

The Adaptive 10 Cycle Goertzel DFT System

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In a previous article entitled “MESA vs Goertzel DFT” we demonstrated that the Goertzel Algorithm, a subset of the Discrete Fourier Transform (DFT), has better frequency detection abilities of sine waves imbedded in noise than the MESA algorithm when the noise amplitude is equal to or greater than the signal amplitude.

MESA which stands for Maximum Entropy Spectral Analysis is more commonly referred to as the Burg AR algorithm in engineering and is a widely used mathematical technique designed to find the frequencies in data. MESA was developed by J.P Burg for his Ph.D dissertation at Stanford University in 1975. The use of the MESA technique for stocks and futures has been written about in many articles.

The Fourier Transform is a mathematical technique named after the famed French mathematician Jean Baptiste Joseph Fourier 1768-1830. It's digital form, namely the discrete-time Fourier Transform series, is a widely used mathematical technique to find the frequency spectrum of discrete time sampled data. The Fast Fourier Transform(FFT) is a computationally efficient algorithm which is designed to rapidly evaluate the discrete-time Fourier Transform series. The use of the DFT has been written about in many articles in this magazine (see references section).

Previous researchers using MESA constrained themselves to using MESA to find only the cycle with the highest amplitude and called that cycle the dominant cycle. In this article we will find and use the frequencies with the ten highest amplitudes to create a noise filtered signal curve that we will follow to create our system buy and sell signals.

The Goertzel Algorithm.

The value of the Discrete Fourier Transform is that for N input data points, the DFT can not only find the frequencies in the data but also the amplitude and phases of the sine waves in the data at the discrete frequency points of $1/N, 2/N, \dots$ to $(N/2)/N$. For instance if N, the number of price bars or closes, were equal to 20 we could find the amplitude and phases for the 20 period cycle (20 bars/ cycle), the 10 period cycle, the $6 \frac{2}{3}$ period cycle all the way down to the 2 period cycle. We could then find the ten cycles with the highest amplitudes and reconstruct a new signal with those ten periods, amplitudes and phases. Remember, cycle= 1/period. As traders we are more accustomed to thinking of terms of periods like the 3 day cycle (frequency = $1/3$ cycle/day) or the 10 bar cycle (frequency = $1/10$ cycles/bar)etc. Unfortunately the DFT can only calculate equally spaced frequencies of $1/N$. In the above example, where $N=20$, using the DFT we could only calculate the amplitudes of the periods (periods= $1/\text{frequency}$) of 20, 10, $6 \frac{2}{3}$ etc. What if the true signal period was between 20 and 10? The DFT couldn't find it. That was the advantage of MESA. MESA was not constrained to the $1/N$ spacing. With MESA any grid of frequencies could be examined. Fortunately, as shown in my previous article, a special subset of the discrete Fourier transform called the Goertzel algorithm can be used to find frequencies in between the $1/N$ frequency divisions. The Goertzel algorithm is used extensively in tone detection. We all are familiar with cellular phones. When you press a button have you ever

wondered how the telephone company knows what button you pushed? The answer is the Goertzel algorithm. This algorithm is built into tiny integrated circuits and immediately (within 43ms) detects the tone of the button you pushed. Here we will use the Goertzel algorithm to detect the frequencies of the price series we give it.

Despite the advantage of the Goertzel algorithm frequency detection abilities, it has three drawbacks. One drawback of the Goertzel algorithm is that it is much slower than the Fast Fourier transform. If we had 512 data points and we wanted to look at 128 different frequencies, the FFT computation would be proportional to $512 \cdot \log_2(512) = 4608$ operations while the Goertzel computation would be proportional $512 \cdot 128 = 65536$ operations. In other words, in this case, the Goertzel algorithm would take more than 10 times longer than the FFT to compute. However, as a computation time comparison, Goertzel would take about half the time as MESA to compute those 128 frequencies. The second drawback of the Goertzel algorithm is that in order to find a frequency in Goertzel when the noise amplitude is high you need enough data so that your lowest frequency (largest period) is able to complete at least 3 cycles. This means that if we were examining daily closing prices and we wanted to find periods of 75 days and less than we would need at least $3 \cdot 75 = 225$ days of prices in order to detect that period in noisy data. The third drawback of the Goertzel algorithm is that while it can detect the frequency within the $1/N$ spacing it cannot detect more than one frequency within that spacing. For instance if $N=20$, and we are looking for a frequency between $1/20$ and $2/20$, the Goertzel can detect the frequency anywhere at or between these two frequencies. However, if there was a second frequency in-between these two frequencies, Goertzel could not find it. If there were two frequencies between these two values, Goertzel would produce one frequency as a weighted average of the two. This is where MESA has a clear advantage. On data with a low noise component, Mesa could detect these two closely spaced frequencies. However, if the data is very noisy than MESA would not be able to detect the closely spaced frequencies either.

The Adaptive 10 Cycle Goertzel-DFT System.

The nature of intraday price movements are constantly changing due to current economic surprises, events and trader sentiment. Also the time of year changes the nature of intraday markets, such as the seasons, holidays, vacation time, etc. As such, the periods or frequencies found on intraday prices 3 weeks ago may no longer be the same as the frequencies found on today's intraday data. We would expect the frequencies found on intraday data to vary over time.

For this system we will create an indicator that walks forward one bar at a time. The indicator will take a fixed number, N , of closing prices and use the Goertzel Algorithm(GA) to find the ten frequencies with the highest amplitudes. Using those ten frequencies, amplitudes and phases, we will construct a new price that forecasts the price one bar ahead. We will save this next bar forecast value, or forecast point. Next we will move the N closing prices forward one bar adding the next bar and dropping the last bar so that we have exactly N closing prices again. With this new N closing prices window we compute the next bar forecast value and save it. We keep marching the N closing price window forward 1 bar at a time, calculate and save the new forecast point until we reach the end of our data. We will then connect all the generated forecast points to produce a curve that creates the ten cycle next bar forecast as the ten frequencies used to create

the next bar forecast change over time. Thus this curve adapts to the closing price's changing frequencies and projects one day ahead so that its lag is minimized when things change.

For example suppose we had daily data from 7/15/98 to 7/20/98. We would calculate and save the next day forecast value of 7/20/98 and slide our data window up one day from 7/16/98 to 7/21/98. We would calculate the next day's forecast value of this new data window and save the forecast value of 7/21/98. We would keep sliding our data window forward one day at a time, calculate the next day's forecast value and save that forecast point. When we reached the end of our data, we would then connect all these saved forecast points to create a new curve which we would follow and use to create the system buy and sell signals.

Adaptive 10 Cycle DFT System Construction Details

Unfortunately constructing the 10 cycle DFT of a price data series is not quite as simple as just taking N closing prices, and directly plugging them into a Goertzel algorithm.

The DFT assumes the time domain sample is periodic and repeats. Suppose a price series starts at 400 and wiggles and wags for 512 data samples ending at the value of 600. The DFT assumes that the price series starts at zero, suddenly jumps to 400, goes to 600 and suddenly jumps down to zero again and then repeats. The DFT must create all kinds of different frequencies in the frequency domain to try and match this type of behavior. These false frequencies created to match the jumps and the high average price completely swamp the amplitudes of any real frequencies making them look like noise. Fortunately this effect can be almost eliminated by a simple technique called end point flattening.

The calculation of end point flattening coefficients is simple. If $x(1)$ represents the first price in the sampled data series, $x(n)$ represent the last point in the data series and $y(i)$ equal to the new endpoint flattened series then:

$$a = x(1) \quad b = (x(n) - x(1)) / (n - 1)$$

$$y(i) = x(i) - [a + b * (i - 1)] \quad \text{for } i = 1 \text{ to } n \quad (1)$$

We can see that when $i=1$ then $y(1)=0$ and when $i=n$ then $y(n)=0$. What we've done is subtract the beginning value of the time series to make the first value equal to zero and then rotate the rest of the time series such that the end point is now zero. This technique reduces the endpoint distortion but introduces a low frequency artifact into the Fourier Frequency spectrum. Fortunately we won't be looking for frequencies in that range so this distortion will have minimal impact.

10 Cycle Goertzel-DFT Curve Construction

Before we start we have to determine what is the largest period we will be looking to include in the 10 cycle DFT construction. For intraday data the 3 day cycle is very important. Higher periods are also important but they don't impact on intraday trading as much. For this article we will use 5 minute bars of the E-Mini. The E-Mini consists of 81 five minute bars a day. Thus

the 3 day cycle would need 243 bars. In order to detect the 3 day cycle we need the cycle to repeat at least three times. This means we need at least $3*243 = 729$ data points in order to detect the 3 day cycle using 5 minute bars. Ten days of 5 min bars is 810 bars. We will use 810 five min bars of E-Mini data.

For this article a sliding time-bar window of 810 five minute bars of the E-Mini Mar/03 futures from 1/2/03 to 3/21/03 will be used. The first time window will be the 810 price bars from 1/2/03 through 1/15/03.

We will use the Goertzel algorithm described in our previous article to determine the amplitude of the frequencies. We will also use the Goertzel algorithm to determine the phases of the frequencies with the 10 highest amplitudes.

Step 1 End flatten the 810 prices using equation (1) above.

Step 2 Use the Goertzel algorithm to calculate the amplitudes for the frequencies of $1/270$ down to $1/6$. Frequency = $1/\text{period}$. We are scanning for periods because as traders we think in terms of periods not frequencies. That is, the 3 day cycle, the 20 bar cycle etc. Thus, here we are scanning for frequencies of 270 bars/cycle to 6 bars/cycle. The frequencies of 5bars/cycle down to 2 bars/cycle move to fast with 5min bars to take advantage trading these cycles. The slippage and commissions would eat up any profits made.

Step 3 Find the ten frequencies with the highest amplitudes, calculate the phases of these frequencies and save these amplitudes ($a[i]$), phases ($\phi[i]$) and frequencies ($f[i]$). Where $[i]$ is one of the ten highest amplitudes found.

Step 4 Calculate the forecast next bar value and the end point bar value using the above ten frequencies, amplitudes and phases. The forecast(fp) = $\sum_{i=1}^{10} a[i] * \cos(2 * \text{PI} * f[i] * 811 + \phi[i])$ and the Endpoint(ep) = $\sum_{i=1}^{10} a[i] * \cos(2 * \text{PI} * f[i] * 810 + \phi[i])$ where $a[i]$, $f[i]$ and $\phi[i]$ are the ten frequency, amplitude and phases found.

Step 5 Save the calculated forecast next bar point and the end point values. Call the forecast next point $fp(k)$ and the end point $ep(k)$ where k is the denotes the order of the sliding window. That is, the first sliding window $k=1$, the second, $k=2$, etc. Slide the 810 bar data window forward one bar, and repeat steps 1 through 4.

Why do we need fp and ep ? When the data window is moved forward one bar at a time a new data sample is added to the end and the data sample at the beginning is subtracted. This adding and subtracting causes the end point flattening coefficients and the power in the frequency spectrum to jump around creating distortion and jitter in the calculation of the forecast next bar point. This random jumping as the data window slides forward in time adds a small random jump to the forecast next bar point curve. Fortunately this jumping can be minimized by creating a curve from the two saved end points, $fp(k)$ and $ep(k)$, above in step 5. Since turning points

are of interest rather than magnitude then in **step 5** a new variable will be created called **sumv** where

$$\text{sumv}(k) = \text{sumv}(k-1) + \text{fp}(k) - \text{ep}(k)$$

$\text{fp}(k) - \text{ep}(k)$ is like a one bar ahead momentum or velocity. This new curve **sumv(k)** is the sum of all the changes in the next bar's ten cycle forecast value $\text{fp}(k)$ from the end point ten cycle value. This change series minimizes the magnitude jump problem creating a fairly smooth momentum sum curve.

The Adaptive 10 Cycle Goertzel-DFT System Defined

Even though **sumv** is a fairly smooth curve as seen in on the charts, it still has a number of short term wiggles preventing us from simply going long when the curve turns up and going short when the curve turns down. To create a system, we will use a simple curve following technique.

Buy Rule:

- **IF sumv** has moved up by more than the point amount of *pntup* from the lowest low recorded in **sumv** while short then buy the ES Mar/03 futures at the market..

Sell Rule:

- **IF sumv** has moved down by more than the point amount *pntdn* from the highest high recorded in **sumv** while long then sell the ES Mar/02 futures at the market.

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Intraday Bars Exit Rule:

Close the position 15 minutes before the E-Mini close when stock trading stops (no trades will be carried overnight).

Intraday Bars First Trade of Day Entry Rule:

Ignore all trade signals before 10:30 EST (60 minutes after the open). (explanation below)

Walk Forward Optimization

The walk forward procedure will be applied as follows. A test period of 5 weeks plus 10 days of the ES Mar/02 5 minute bar data, January 2, 2003 through February 21, 2002, is chosen and system parameter values are found through optimization on this intraday data segment. The parameter values found are then applied to the out-of-sample 5 minute intraday bar data following the test segment which in this case is the two weeks from February 24th, 2003 to March 7th, 2003. In the System Buy and Sell rules above we have included a first trade of the day entry rule. We've included this rule because often there are gaps in the open creating immediate system buys and sells. Many times these gaps are closed creating a losing whipsaw trade. In order to avoid the opening gap whipsaw trade problem we've delayed the first trade of the day for 60 minutes until after 10:30 EST.

Why a 5 week intraday data test segment? There is no correct ratio of test data needed to produce good one week intraday out-of-sample results. By experimenting with different window lengths, the five to one ratio seemed to work well. In walk forward testing, enough data is

needed to model most of the price dynamics that will be encountered in the out-of-sample segment, but not so much data that when the price dynamics start to change they are swamped by the weight of distant past data price dynamics that no longer apply. It is important to get good results over time in walk forward testing because if you can not get good results in the out-of-sample segments, then the price dynamics cannot be modeled with the system . This means that real time performance will not match the optimized curve fitted performance results. Traders observe this type of random performance (that is it looks great on paper but falls apart in real time) when trying systems based on optimization curve fitting or anecdotal “proof” (looking at 3 or 4 successful cases only) without any out-of-sample testing .

Finding The System Parameters Using Walk Forward Optimization

There are two system parameters to find *pntup*, and *pntdn*. The best parameters will be defined as those values that give the best average Net Profits with the maximum winning bars, minimum losing bars, minimum drawdown, minimum largest losing trades. In addition, the results should be stable, e.g. the profits, wins, and drawdowns should not change by much as the parameters move by a small amount away from their optimum values. Also in choosing the “best” parameters, only those parameters sets whose maximum consecutive losses were 4 or less were considered. Optimization is defined as the search for the parameter values that give the best results as defined above. It should be noted that in this stage of system development, the only thing indicated by the optimum values that are found in the test portion is that the data has been *curve fitted* as best it can with this system. Without further testing on out-of-sample data there is no way to tell if the system will work in the future.

Almost any real time series or even a random time series defined over a fixed number of bars can be curve fitted rather easily. The performance results and the statistical measurements that validate this performance of the curve fit will look excellent giving the false illusion of future profitability. However, these excellent performance and associated statistics on the test section in no way validate how the system will perform on data it has not been optimized on. Only out-of-sample testing, that is testing on data the parameters were not derived on, can determine if the parameters found in the test section have captured the price dynamics.

Results

Figure 4 presents a table of the test window optimum parameters for the E-Mini Mar/03 5min data series.

Start Date	End Date	Pntup	Pntdn
01/02/03	02/21/03	2.00	1.00

Figure 4 Optimum Parameter Values For Test Data Segment

Figures 5 presents the performance summary of the test segment using the optimum parameters shown in Figure 4.

Figure 6 presents the performance summary of the out-of-sample data segment from 02/24/03 to 03/17/03. This performance represents what would have happened in *real time* if one used the parameters found in the test section. Slippage, and commissions are not included.

Figure 7 presents a trade by trade summary for the out-of-sample trades from 02/24/03 to 03/07/03.

Figures 8A through 8C present the out-of-sample 5 minute bar charts of the E-Mini Mar/03 futures with the **Adaptive 10 Cycle Goertzel DFT** Curve and all the buy and sell signals from the trade by trade summary of Figure 7 indicated on the charts.

Discussion of System Performance

As can be observed from the test sample Performance summary in Figure 5 and the out-of-sample performance summary of Figure 6, the out-of-sample “All Trades” performance was similar to the test sample “All Trades” performance. This similar performance in average winning and losing trades and drawdowns indicate that five weeks of test data was enough to capture the intraday price dynamics of the E-Mini Mar/03 contract for two weeks into the future.

Observing the out-of-sample trade by trade summary of Figure 7, we can see that the system did better on the short trades than it did on the long trades. The average number of bars in short winning trades was 38 and the average number of bars in long winning trades was 26. This is indicative of a down market during the out-of-sample dates.

In observing the charts, we can see that the system did very well in catching every major intraday trend. Overall the **Adaptive 10 Cycle DFT** system did a good job in minimizing the losses due to the inevitable whipsaws that will occur in any trading system and maximizing the profits from the major intraday trend moves of the E-Mini futures.

In order to use this system in real time trading, at least ten to twenty more test and out-of-sample windows from the past would have to be examined to gain confidence that the results above were not due to pure chance. Why is all this further testing necessary? Randomness plays a great part in price series. One can never be sure when using price data over a **fixed time interval** whether the good performance numbers from system optimization are from curve fitting the random process, which will not be repeated, or the system optimization has captured the underlying price dynamics. Only testing on out-of-sample data from many trials of in-sample optimization can one become confident that the performance results are not due to pure chance. System trading is a lot like playing poker or bridge or any game that has a random element involved. Any card player, no matter what their skill will have winning streaks. These lucky winning streaks in no way determine if you will be a winner after many card sessions. Only many playing sessions (in our case out-of-sample testing) will determine if you can be net winner by skill.

References:

1. Burg, J. P., ‘Maximum Entropy Spectral Analysis’, Ph.D. dissertation, Stanford University, Stanford, CA. May 1975.

2. Goertzel, G., “An Algorithm for the evaluation of finite trigonometric series” American Math Month, Vol 65, 1958 pp34-35.
3. Kay, Steven M., “Modern Spectral Estimation”, Prentice Hall, 1988
4. Marple, Lawrence S. Jr., “Digital Spectral Analysis With Applications”, Prentice Hall, 1987
5. Meyers, Dennis , “MESA vs Goertzel DFT”, *Working Paper*,
<http://meyersanalytics.com/articles.htm>
6. Press, William H., et al, “Numerical Recipes in C++: the Art of Scientific Computing”, Cambridge Press, 2002.
7. Oppenheim, A, Schafer, R. and Buck, J., “Discrete Time Signal Processing”, Prentice Hall, 1996, pp663-634
8. Proakis, J. and Manolakis, D. “Digital Signal Processing-Principles, Algorithms and Applications”, Prentice Hall, 1996., pp480-481

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Figure 5 Test Segment Performance Summary for E-Mini Adaptive 10 Cycle DFT System
ES Mar/03 5 min 01/02/03-02/21/03

Performance Summary: All Trades

Total Net Profit	\$3,725.00	Open position P/L	\$0.00
Gross Profit	\$8,837.50	Gross Loss	(\$5,112.50)
Total # of trades	62	Percent profitable	56.45%
Number winning trades	35	Number losing trades	27
Largest winning trade	\$912.50	Largest losing trade	(\$537.50)
Average winning trade	\$252.50	Average losing trade	(\$189.35)
Ratio avg win/avg los	1.33	Avg trade (win & los)	\$60.08
Max consec. Winners	4	Max consec. losers	3
Avg # bars in winners	30	Avg # bars in losers	22
Max intraday drawdown(\$1,887.50)		Max # contracts held	1
Profit Factor	1.73		

Performance Summary: Long Trades

Total Net Profit	\$1,350.00	Open position P/L	\$0.00
Gross Profit	\$3,500.00	Gross Loss	(\$2,150.00)
Total # of trades	29	Percent profitable	48.28%
Number winning trades	14	Number losing trades	15
Largest winning trade	\$862.50	Largest losing trade	(\$312.50)
Average winning trade	\$250.00	Average losing trade	(\$143.33)
Ratio avg win/avg los	1.74	Avg trade (win & los)	\$46.55
Max consec. Winners	3	Max consec. losers	4
Avg # bars in winners	25	Avg # bars in losers	22
Max intraday drawdown (\$812.50)		Max # contracts held	1
Profit Factor	1.63		

Performance Summary: Short Trades

Total Net Profit	\$2,375.00	Open position P/L	\$0.00
Gross Profit	\$5,337.50	Gross Loss	(\$2,962.50)
Total # of trades	33	Percent profitable	63.64%
Number winning trades	21	Number losing trades	12
Largest winning trade	\$912.50	Largest losing trade	(\$537.50)
Average winning trade	\$254.17	Average losing trade	(\$246.88)
Ratio avg win/avg los	1.03	Avg trade (win & los)	\$71.97
Max consec. Winners	6	Max consec. losers	4
Avg # bars in winners	33	Avg # bars in losers	21
Max intraday drawdown(\$1,412.50)		Max # contracts held	1
Profit Factor	1.80		

**Figure 6 Out-Of-Sample Performance Summary for E-Mini Adaptive 10 Cycle DFT System
ES Mar/03 5 min 02/24/03-03/07/03**

Performance Summary: All Trades

Total Net Profit	\$1,625.00	Open position P/L	\$0.00
Gross Profit	\$2,837.50	Gross Loss	(\$1,212.50)
Total # of trades	23	Percent profitable	56.52%
Number winning trades	13	Number losing trades	10
Largest winning trade	\$587.50	Largest losing trade	(\$262.50)
Average winning trade	\$218.27	Average losing trade	(\$121.25)
Ratio avg win/avg los	1.80	Avg trade (win & los	\$70.65
Max consec. Winners	5	Max consec. losers	2
Avg # bars in winners	33	Avg # bars in losers	21
Max intraday drawdown (\$412.50)		Max # contracts held	1
Profit Factor	2.34		

Performance Summary: Long Trades

Total Net Profit	\$600.00	Open position P/L	\$0.00
Gross Profit	\$1,287.50	Gross Loss	(\$687.50)
Total # of trades	11	Percent profitable	45.45%
Number winning trades	5	Number losing trades	6
Largest winning trade	\$587.50	Largest losing trade	(\$250.00)
Average winning trade	\$257.50	Average losing trade	(\$114.58)
Ratio avg win/avg los	2.25	Avg trade (win & los	\$54.55
Max consec. Winners	2	Max consec. losers	3
Avg # bars in winners	26	Avg # bars in losers	20
Max intraday drawdown (\$687.50)		Max # contracts held	1
Profit Factor	1.87		

Performance Summary: Short Trades

Total Net Profit	\$1,025.00	Open position P/L	\$0.00
Gross Profit	\$1,550.00	Gross Loss	(\$525.00)
Total # of trades	12	Percent profitable	66.67%
Number winning trades	8	Number losing trades	4
Largest winning trade	\$350.00	Largest losing trade	(\$262.50)
Average winning trade	\$193.75	Average losing trade	(\$131.25)
Ratio avg win/avg los	1.48	Avg trade (win & los	\$85.42
Max consec. Winners	6	Max consec. losers	2
Avg # bars in winners	38	Avg # bars in losers	22
Max intraday drawdown (\$425.00)		Max # contracts held	1
Profit Factor	2.95		

FIGURE 7 Out-Of-Sample Trade By Trade Summary
E-Mini Mar/03 5min Bars Adaptive 10 Cycle DFT System 02/24/03-03/07/03

Entry Date	Entry Time		Entry Price	Exit Date	Exit Time	Exit Price	Bars InTrd	Trade \$P&L	Trade %P&L	Trade Max\$Pft	Time	Trade Max\$DD	Time
2/24/03	1035	Sell	839.50	2/24/03	1430	832.50	47	\$350	0.83%	\$388	1235	(\$50)	1055
2/24/03	1430	Buy	832.50	2/24/03	1600	832.00	18	(\$25)	-0.06%	\$200	1505	(\$50)	1430
2/25/03	1035	Sell	821.00	2/25/03	1405	826.25	42	(\$263)	-0.64%	\$100	1220	(\$263)	1400
2/25/03	1405	Buy	826.25	2/25/03	1600	838.00	23	\$588	1.42%	\$650	1550	\$0	1405
2/26/03	1145	Sell	831.75	2/26/03	1345	832.75	24	(\$50)	-0.12%	\$125	1150	(\$50)	1340
2/26/03	1345	Buy	832.75	2/26/03	1510	827.75	17	(\$250)	-0.60%	\$38	1425	(\$313)	1500
2/26/03	1510	Sell	827.75	2/26/03	1600	826.25	10	\$75	0.18%	\$75	1600	(\$88)	1550
2/27/03	1035	Buy	834.00	2/27/03	1310	837.00	31	\$150	0.36%	\$375	1110	\$0	1035
2/27/03	1310	Sell	837.00	2/27/03	1600	836.50	34	\$25	0.06%	\$275	1455	(\$150)	1340
2/28/03	1035	Buy	844.00	2/28/03	1140	846.50	13	\$125	0.30%	\$125	1140	(\$38)	1050
2/28/03	1140	Sell	846.50	2/28/03	1600	841.00	52	\$275	0.65%	\$438	1545	\$0	1140
3/3/03	1035	Buy	846.00	3/3/03	1115	841.25	8	(\$238)	-0.56%	\$38	1035	(\$263)	1105
3/3/03	1115	Sell	841.25	3/3/03	1600	834.50	57	\$338	0.80%	\$450	1535	(\$138)	1235
3/4/03	1035	Sell	828.75	3/4/03	1255	827.75	28	\$50	0.12%	\$100	1115	(\$100)	1045
3/4/03	1255	Buy	827.75	3/4/03	1430	825.00	19	(\$138)	-0.33%	\$125	1310	(\$138)	1425
3/4/03	1430	Sell	825.00	3/4/03	1600	822.00	18	\$150	0.36%	\$163	1555	(\$13)	1430
3/5/03	1035	Buy	824.50	3/5/03	1420	824.25	45	(\$13)	-0.03%	\$213	1255	(\$263)	1100
3/5/03	1420	Sell	824.25	3/5/03	1530	827.25	14	(\$150)	-0.36%	\$113	1445	(\$200)	1505
3/5/03	1530	Buy	827.25	3/5/03	1600	829.50	6	\$113	0.27%	\$113	1600	(\$50)	1535
3/6/03	1035	Sell	828.00	3/6/03	1510	822.25	55	\$288	0.69%	\$400	1300	(\$50)	1035
3/6/03	1510	Buy	822.25	3/6/03	1600	821.75	10	(\$25)	-0.06%	\$100	1515	(\$63)	1530
3/7/03	1035	Sell	820.75	3/7/03	1115	822.00	8	(\$63)	-0.15%	\$125	1045	(\$63)	1110
3/7/03	1115	Buy	822.00	3/7/03	1600	828.25	57	\$313	0.76%	\$350	1225	(\$100)	1425

FIGURE 8A E-Mini Mar/03 5min GDFT System 02/24/2003 - 03/07/2003

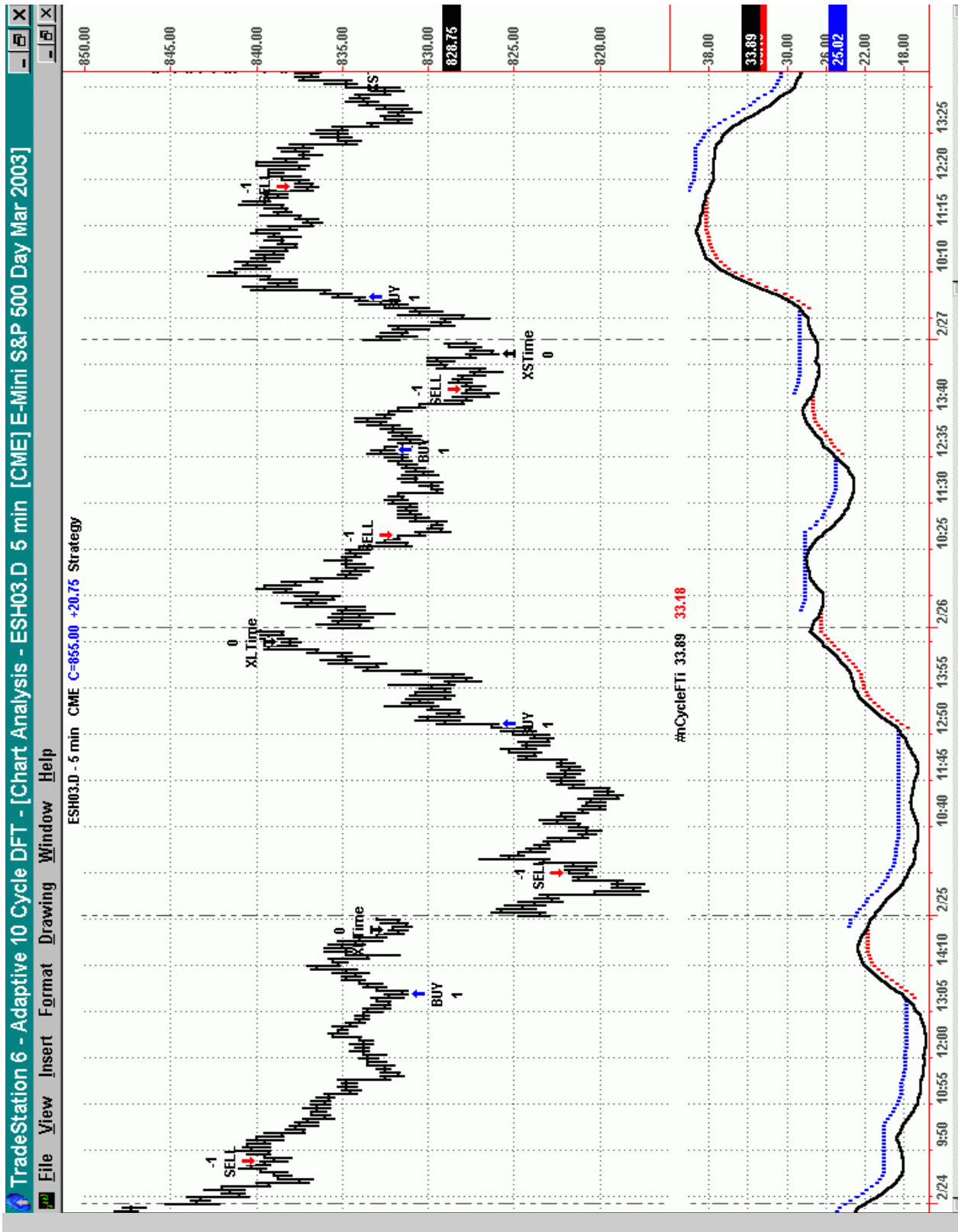


FIGURE 8B E-Mini Mar/03 5min GDFT System 02/24/2003 - 03/07/2003

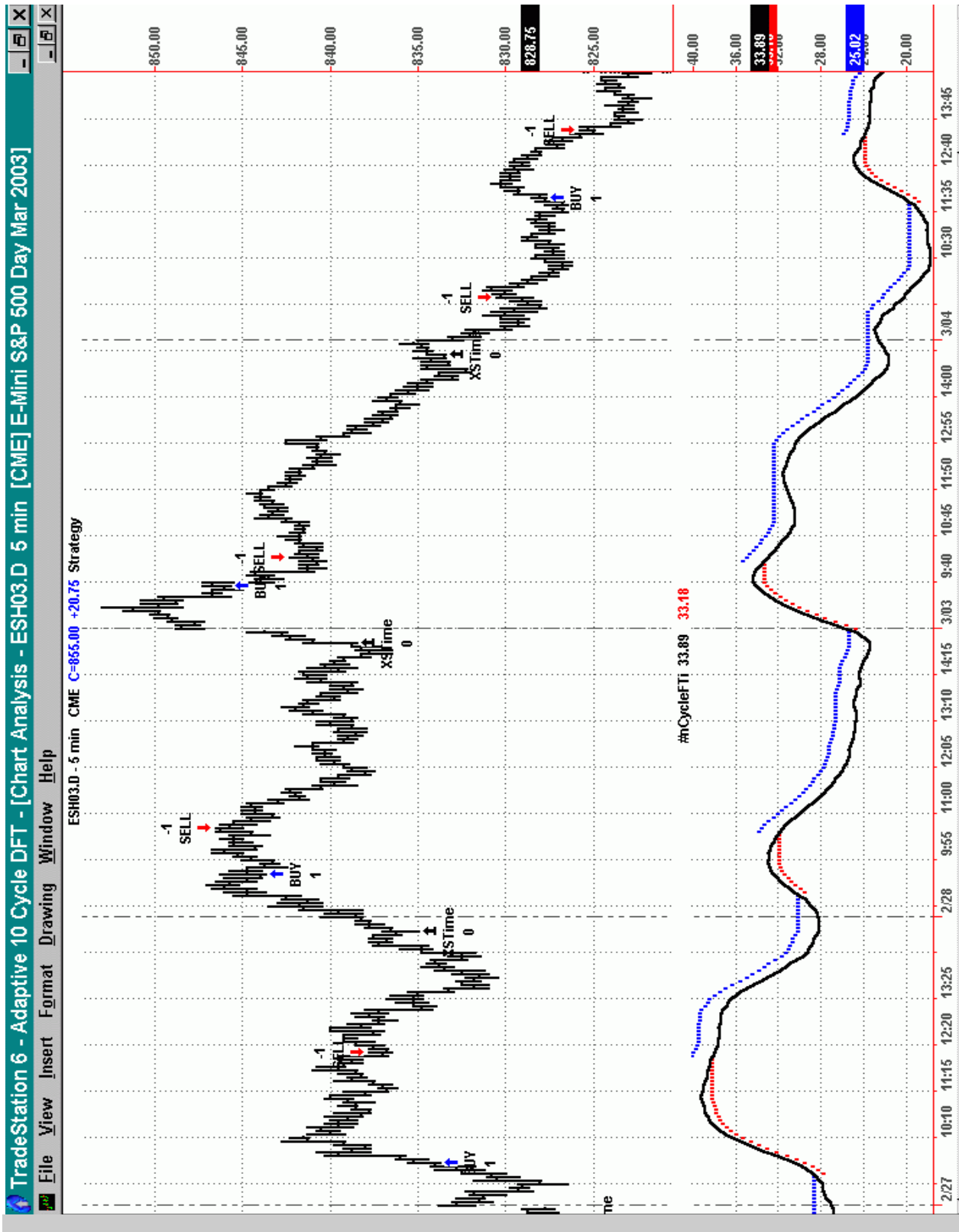


FIGURE 8C E-Mini Mar/03 5min GDFT System 02/24/2003 - 03/07/2003

