

**Trading DIA ETF 5min Bars Using the nth Order Fixed Memory Polynomial Velocity Algorithm**  
**Walk Forward in-sample 20 Trading weekdays and out-of-sample 1 Trading weekday**  
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In previous working papers at <https://meyersanalytics.com/papers> we showed how the application of a price curve generated by the **Nth Order Fixed Memory Polynomial Velocity** could be used to develop a strategy to buy and sell futures and stocks intraday. The reasoning behind this type of strategy was to only trade when the price trend velocity was above a certain threshold. Many times, prices meander around without any notable trend, and this is considered noise. During these times we do not wish to trade because of the cost of whipsaw losses that would occur from this type of price action. When a price trend finally starts, the velocity of that price trend moves above a minimum threshold noise value. Thus, the velocity strategy would only issue a trade when certain velocity thresholds above “noise” levels are crossed.

The velocity strategy that we will use here to trade the Dow Jones ETF (**DIA**) is called the nth Order Polynomial Velocity Strategy. The nth Order Adaptive Polynomial Velocity Strategy has several unknown inputs that we must determine before we can use this strategy to trade. These unknown inputs are the polynomial order or degree, the optimum number of lookback prices we need to determine the coefficients of the polynomial and finally the velocity thresholds. Here we will use Walk Forward Optimization and out-of-sample testing to determine the “best” polynomial inputs as well as how these inputs change over time. We will use the nth Order Fixed Memory Polynomial Velocity Strategy to trade the DIA ETF on an intraday basis using 5-min bar price data from 12/1/2019 to 7/9/2021.

**The nth Order Fixed Memory Velocity Strategy Defined**

The least squares forecast nth order fixed memory polynomial velocity is constructed by solving for the coefficients  $\beta_0, \beta_1, \beta_2, \beta_3 \dots \beta_n$  for the discrete orthogonal Legendre polynomials each day using the last **N bars** of closing prices and the equation for  $\beta_j$  shown in the “Math” appendix at the end of this working paper. Then nth Order Fixed Memory Polynomial **Velocity(T+1)** is constructed from the equation shown in the “Math” appendix.

Due to polynomial mathematics, the Velocity of the 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> order degree polynomial curve changes faster than it’s corresponding first order degree polynomial velocity. Whether higher order polynomial velocities are an advantage or not, will be determined by the computer when we use a walk forward optimization technique described below.

At each bar we calculate the nth order degree (1<sup>st</sup> through 4<sup>th</sup>) fixed memory polynomial velocity from the formulas in the “Math” appendix. As will be shown below, walk forward optimization will determine the **degree** for the nth order polynomial velocity, the number of lookback prices, **N**, needed to compute the polynomial coefficients and the threshold amounts **vup** and **vdn**. When the nth order degree velocity is greater than the threshold amount **vup** we will go long. When the velocity is less than the threshold amount **-vdn** we will go short.

**Buy Rule:**

**IF Velocity** is greater or equal than the threshold amount **vup and Velocity[1] is less than vup** then buy at the market.

**Sell Rule:**

**IF Velocity** is less than or equal than the threshold amount **-vdn and Velocity[1] is greater than -vdn** then sell at the market.

Where Velocity[1] is the velocity on the previous bar.

#### **Intraday Bars Exit Rule:**

Close the position at 1555 EST (No trades will be carried overnight).

#### **Testing The Polynomial Velocity Strategy Using Walk Forward Optimization**

There will be four strategy parameters to determine:

1. **degree**, degree=1 for straight line velocity, degree=2 for 2<sup>nd</sup> order velocity, etc.
2. **N**, is the number of lookback bars of prices to calculate the **velocity**.
3. **vup**, the threshold amount that velocity must be greater than to issue a buy signal
4. **vdn**, the threshold amount that velocity must be less than to issue a sell signal

As mentioned, to test this Strategy we will use five-minute bar prices of the Dow Jones ETF traded on the NYSE and known by the symbol DIA for the 399 trading days from December 9, 2019, to July 9, 2021.

We will test the FixmVn strategy with the above DIA ETF 5 min bars on a **walk forward basis**, where the in-sample (**IS**) will be 20 trading weekdays and the out-of-sample (**OOS**) will be the next trading weekday following as will be described below.

#### **What Is a Walk Forward Optimization with In-Sample Section and Out-Of-Sample Sections?**

Whenever we do a TradeStation or Multicharts (TS/MC) optimization on many different strategy inputs, TS/MC generates a report of performance metrics (total net profits, number of losing trades, etc.) vs these different strategy inputs. If the report is sorted on say the total net profits(**tnp**) performance metric column then the highest **tnp** would correspond to a certain set of inputs. This is called an **in-sample (IS) section**. If we choose a set of strategy inputs from this report based upon some performance metric, we have no idea whether these strategy inputs will produce the same results on future price data or data they have not been tested on. Price data that is not in the in-sample section is defined as **out-of-sample data**. Since the performance metrics generated in the in-sample section are mostly due to "curve fitting" or "data mining" it is important to see how the strategy inputs chosen from the in-sample section perform on out-of-sample data.

What do we mean by "**curve fitting**" or **data mining**? As a simple example, suppose you were taking a subway to work. In the subway car you are in, suppose you counted the number of blond women in that car and suppose the percent of blond women vs all other women hair colors was 80%. Being that you cannot observe what is in the other subway cars, you would assume that all the other subway cars and perhaps all women had the same percentage of blond hair. This observation was due to chance. That is an example of curve fitting. The same goes for combinatorial searches. You are observing results from a finite sample of data without knowing the data outside the sample you examined.

Walk forward analysis attempts to minimize the curve fitting of price noise by using the law of averages from the Central Limit Theorem on the out-of-sample performance. In walk forward analysis the data is broken up into many in-sample and out-of-sample sections. Usually for any strategy, one has some performance metric selection procedure, which we will call a **filter**, used to select the strategy input parameters from the optimization run. For instance, a **filter** example might be all cases that have a profit factor (PF) greater than 1 and less than 3. For the number of cases left, we might select the cases that had the best percent profit. This procedure would leave you with one case in the in-sample section output and its associated strategy input parameters. Now suppose we ran our optimization on each of our many in-sample sections and applied our **filter** to each in-sample section output. We would then use the strategy input parameters found by the **filter** in each in-sample section on the out-of-sample section immediately following that in-sample section. The input parameters found in each in-sample section and applied to each out-of-sample section would produce independent net profits or losses for each of the out-of-sample sections. Using this method, we now have "x" number of independent out-of-sample section profit and losses from our **filter**. If we take the average of these out-of-sample section net profits and losses, then we will have an estimate of how our strategy will perform on average. Due to the Central Limit Theorem, as your

sample size increases, the spurious noise results in the out-of-sample section performance tend to average out to zero in the limit, leaving us with what to expect from our strategy and filter. **Mathematical note: This assumption assumes that the out-of-sample returns are from probability distributions that have a finite variance.**

Why use the walk forward technique? Why not just perform an optimization on the whole price series and choose the input parameters that give the best total net profits or profit factor or some other performance metric? Surely the price noise cancels itself out with such a large number of in-sample prices and trades. Unfortunately, nothing could be farther from the truth! Optimization is a misnomer and should really be called combinatorial search. As stated above, whenever we run a combinatorial search over many different combinations of input parameters on noisy data on a fixed number of prices, **no matter how many**, the best performance parameters found are guaranteed to be due to **“curve fitting”** the noise and signal. The price series that we trade consists of random spurious price movements, which we call noise, and repeatable price patterns (*if they exist*). When we run, for example, 5000 different inputs parameter combinations, the best performance parameters will be from those strategy input variables that are able to produce profits from the price pattern **and** the random spurious movements. While the price patterns will repeat, the same spurious price movements will not. If the spurious price movements that were captured by a certain set of input parameters were a large part of the total net profits, as they are in real intraday price series, then choosing these input parameters will produce losses when traded on future data. These losses occur because the spurious price movements will not be repeated in the same way. This is why strategy optimization or combinatorial searches, also called back testing, with no out-of-sample testing cause losses when traded in real time from something that looked great in the in-sample section.

To gain confidence that our input parameter selection method or filter, using the optimization output of the in-sample data, will produce profits, we must test the input parameters we found in the in-sample section on out-of-sample data. In addition, we must perform the in-sample/out-of-sample analysis many times. Why not just do the out-of-sample analysis once or just 10 times? Well just as in Poker or any card game, where there is considerable variation in luck from hand to hand, walk forward out-of-sample analysis give considerable variation in out-of-sample profit “luck”. That is, by pure chance we may have chosen some input parameter set that did well in the in-sample section data **and** the out-of-sample section data. In order to minimize this type of “luck”, statistically, we must repeat the walk forward out-of-sample (**OOS**) analysis over many (>50) in-sample/out-of-sample sections and take an average over all out-of-sample sections. This average gives us an expected out-of-sample return and a standard deviation of out-of-sample returns which allows us to statistically estimate the expected equity and its range for N out-of-sample periods in the future

### **Finding The FixmVn Strategy Parameters Using Walk Forward Optimization**

There are four strategy parameters to find,  $pw$ ,  $N$ ,  $vup$ ,  $vdn$ .

For the test data we will run the TS or MC optimization engine on **DIA** 5 min price bars from 12/9/2019 to 7/9/2021 with the following optimization ranges for the FixmVn strategy inputs. This will create **399, 20 weekday in-sample periods each followed by a 1 day out-of-sample period** (See Figure 1 for the in-sample/out-of-sample periods). The days are weekdays only. Weekdays where the OOS falls on an exchange holiday or partial days are eliminated. Holidays that fall on a weekday create a 19-day **IS**. All other **IS** periods consist of 20 trading weekdays. The optimization ranges are:

1. **pw=degree from 1 to 3**
2. **N from 4 to 16 in steps of 2.**
3. **vup from 0.25 to 3.5 steps of 0.25**
4. **vdn from 0.25 to 3.5 in steps of 0.25**
5. **Mult = 16, iNorm=1 (See Appendix 3, the Normalization Multiplier)**

The above  $pw$ ,  $n$ ,  $vup$ ,  $vdn$  will produce 4116 different input combinations or cases of the strategy input parameters for each of the 399 in-sample/out-of-sample files for the 19 months of 5 min bar DIA data.

### Finding the Best Set of Strategy Inputs to use with an in-sample Metric Filter.

The PWFO generates a number of performance metrics in the in-sample section. (Please see <http://meyersanalytics.com/Walk-Forward-Optimization.html> for a listing of these performance metrics). The question we are attempting to answer statistically, is which performance metric or combination of performance metrics (which we will call a *filter*) applied to a given set of strategy inputs in the *in-sample* section will produce statistically valid profits in the sum of all out-of-sample sections. In other words, we wish to find the best set of strategy inputs *with a metric filter applied* in each *in-sample* section that gives the “best” total out-of-sample results over all out-of-sample sections. This means if we applied our *metric filter* to the strategy inputs chosen in the in-sample section, we would *only trade using those set of strategy inputs* in the next out-of-sample section if the in-sample *metric filter* satisfied our criteria. *Else no trades would be made* in the next out-of-sample section.

### **The Walk Forward Strategy – Strategy Inputs with Metric Filters Explorer.**

We wish to find *one* set of strategy inputs that we can trade in every out-of-sample section, but we will only trade that set of strategy inputs in the out-of-sample section if and only if they satisfy our in-sample *metric-filter*. Else we will not trade the next out-of-sample section. In this paper the in-sample section is 20 trading days, and the out-of-sample section is the next trading day. After running the PWFO on the in-sample data, we examine the in-sample metric filter that we chose. If the strategy inputs we selected satisfy the in-sample metric filter requirements then we use those strategy inputs to trade the next day. If the strategy inputs do not satisfy the in-sample metric filter we do not trade the next day.

Let us define the in-sample *metric-filter* we will use here: in-sample Profit Factor ( $PF \leq x$ ) and/or Losers in a row ( $lr \leq y$ ), and/or equity curve straight line correlation coefficient ( $r^2(R2) \leq z$ ). That is  $PF \leq x$  and/or  $lr \leq y$  and/or  $R2 \leq z$ .

What we are going to do here is look at every combination in the in-sample section of each **strategy input** with  $PF \leq x$  and/or  $lr \leq y$  and/or  $R2 \leq z$ . This will produce seven **strategy input | metric-filter** combinations:

1. **strategy input |  $PF \leq x, lr \leq y, R2 \leq z$  |**
2. **strategy input |  $PF \leq x, lr \leq y$  |**
3. **strategy input |  $PF \leq x, R2 \leq z$  |**
4. **strategy input |  $PF \leq x$  |**
5. **strategy input |  $LR \leq y, R2 \leq z$  |**
6. **strategy input |  $lr \leq y$  |**
7. **strategy input |  $R2 \leq z$  |**
8. **strategy input – we also examine inputs with no filter**

If the **strategy input | metric-filter** satisfies **the metric-filter** condition in the in-sample section, then we will use those strategy inputs to trade in the out-of-sample section. If not, then there will be no trades in the out-of-sample section.

We will look at all **metric-filter** combinations of  $PF \leq 2$  to 6 step 1,  $LR \leq 3, 5$  step 2 and  $R2 \leq 70$  to 90 step 5. We will also look at the strategy input with no metric-filter. With 4116 different strategy input combinations this will give us 444527 **strategy input | metric-filter** combinations. Each one of these 444527-**strategy input | metric-filter** combinations will be applied to each in-sample section and their out-of-sample performance will be tabulated for all 399 PWFO files.

Below is a snippet of the output from a run of all 444527 combinations sorted by **tONP = total OOS net profit for each strategy input | metric-filter** combination. *The column definitions are defined in Figure 3 below.* This example shows a partial output file from the WFINP program run on the PWFO files generated with the FixmVn that was run on 100 shares of DIA ETF 5-minute bars 399 days from 12/9/2019 to 7/9/2021. The in-sample (IS) period is 20 trading weekdays, and the out-of-sample (OOS) period is 1 trading weekday. This strategy traded between 9am to 1600pm Exchange Time (EST).

From this run, we chose the filter on row 4 of the Figure below. That is,

**3|6|2.25|2.75|0|1555|16|pf<4|lr<5|r2<85.** This is constructed as follows. For the strategy inputs **3|6|2.25|2.75|0|1555|16|** only those in-sample sections that have a **pf ≤ 4** and **lr ≤ 5** and **r2 ≤ 85** are used to trade in the following out-of-sample sections. If the in-sample **pf > 4** and or **lr > 5** and or **r2 > 80** then the out-of-sample section following the in-sample section **is not** traded.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA
1	DIA5mFixm20x1dxo	s12/31/19	e07/09/21	#399	AnyTnp						ISnt2			a(12.5)	s12.5	f444527					c=\$4						
2	pw N vup vdn xop xt mult <PF<LR<R2	toGP	TONP	aoGP	aoTr	aoWT	std	skew	kur	t	oW ol	%Wtr	%P	LLtr	LLp	eqDD	wpr	lpr	#	V20	Dev^2	KTau	eqR2	Blw	BE	tkr bl	Prob
3	3 6 2.5 3 0 1555 16 pf<2 r2<70	13269	11389	85	28.2	3	363	-0.515	6.1	2.92	1.33	51	65	-706	-1437	-3768	11	4	156	19	1114	94	92	52	196	248	3.49E-12
4	3 6 2.25 2.75 0 1555 16 pf<4 lr<5 r2<85	12537	11309	83	40.8	2	226	0.186	4.59	4.52	1.5	56	65	-692	-692	-985	8	4	151	17	675	95	97	45	82	1119	1.20E-12
5	3 6 2.25 2.75 0 1555 16 pf<4 lr<5 r2<90	12618	11194	76	35.4	2.1	232	-0.107	4.93	4.2	1.45	55	65	-692	-766	-1166	8	4	167	17	694	94	97	61	95	658	8.92E-11
6	3 6 2.75 3 0 1555 16 pf<2	12741	10961	81	28.6	2.8	328	-0.613	7.44	3.09	1.43	50	62	-706	-1437	-2762	13	3	158	11	1322	93	89	52	175	280	2.57E-11
7	3 6 2.25 2.75 0 1555 16 pf<4 lr<5 r2<80	12071	10883	84	40.6	2.1	228	0.172	4.59	4.41	1.52	56	65	-692	-692	-985	8	4	144	17	656	94	97	45	86	1037	8.09E-13
8	3 6 2.75 3 0 1555 16 pf<2 r2<90	12584	10828	81	28.7	2.8	331	-0.612	7.32	3.06	1.44	50	62	-706	-1437	-2762	13	3	155	11	1286	93	89	52	179	271	1.98E-11
9	3 6 2.25 2.75 0 1555 16 pf<4 lr<5	12160	10664	69	32.5	2.1	231	-0.069	4.86	3.98	1.38	55	65	-729	-766	-1342	8	4	175	17	676	93	96	66	106	511	1.93E-09
10	3 6 2.75 3 0 1555 16 pf<5 r2<75	12300	10404	74	25.9	2.9	321	-0.642	7.49	2.98	1.35	50	60	-706	-1437	-2762	13	4	166	11	1380	92	87	51	189	246	8.21E-10
11	3 6 2.75 3 0 1555 16 pf<2 r2<75	12300	10404	74	25.9	2.9	321	-0.642	7.49	2.98	1.35	50	60	-706	-1437	-2762	13	4	166	11	1380	92	87	51	189	246	8.21E-10
12	3 6 2.75 3 0 1555 16 pf<6 r2<75	12300	10404	74	25.9	2.9	321	-0.642	7.49	2.98	1.35	50	60	-706	-1437	-2762	13	4	166	11	1380	92	87	51	189	246	8.21E-10
13	3 6 2.75 3 0 1555 16 pf<4 r2<75	12208	10332	74	26	2.9	323	-0.642	7.42	2.96	1.36	50	60	-706	-1437	-2762	13	4	164	11	1357	92	87	51	192	241	6.98E-10
14	3 6 2.75 2.75 0 1555 16 pf<6 lr<5 r2<75	11366	10266	82	41.3	2	245	0.912	6.31	3.93	1.76	52	63	-538	-489	-1393	11	4	139	20	808	92	95	44	109	718	2.16E-12
15	3 6 2.75 2.75 0 1555 16 pf<5 lr<5 r2<75	11366	10266	82	41.3	2	245	0.912	6.31	3.93	1.76	52	63	-538	-489	-1393	11	4	139	20	808	92	95	44	109	718	2.16E-12

### Bootstrap Probability of Filter Results.

Using modern "Bootstrap" techniques, we can calculate the probability of obtaining our filter's total out-of-sample **net** profits by chance. Here is how the bootstrap technique is applied. Suppose as an example, we have 500 files of in-sample/out-of-sample data. A mirror random filter is created. Instead of picking an out-of-sample net profit (OSNP) from a filter row as before, the mirror filter picks a **random** row's OSNP in each of the 500 files. We repeat this random picking in each of the 500 files 5000 times. Each of the 5000 mirror filters will choose a random row's OSNP of their own in each of the 500 files. At the end, each of the 5000 mirror filters will have 500 **random** OSNP's picked from the rows of the 500 files. The sum of the 500 random OSNP picks for each mirror filter will generate a random total out-of-sample net profit (toNP) or final random equity. The average and standard deviation of the 5000-mirror filter's different random toNPs will allow us to calculate the chance probability of our above chosen filter's toNP. Thus, given the mirror filter's bootstrap random toNP average and standard deviation, we can calculate the probability of obtaining our chosen filter's toNP by pure chance alone. Figure 3 lists the 5000-mirror filter's bootstrap average for our 399 out-of-sample files of **-\$12.5** with a bootstrap standard deviation of **\$12.5**. (Side Note. The average is the average per out-of-sample period. So, the average for the random selection would be the random toNP/399 and the average for the filter would be the filter toNP/# of OOS periods traded or 11309/151=74.9). The probability of obtaining our filters average daily net profit of **74.9** is **1.20<sup>-12</sup>** which is **7.0** standard deviations from the bootstrap average. For our filter, in row 4 above, the expected number of cases that we could obtain by pure chance that would match or exceed **\$74.9** is  $[1 - (1 - 1.20 \times 10^{-12})^{444527}] \sim 444527 * 1.20 \times 10^{-12} = 0.00000533$  where **444527** is the total number of different filters we looked at in this run. This number is much much less than one, so it is improbable that our result was due to pure chance.

### Results

**Figure 1** presents a graph of the equity curve generated by using the filter on the 399 days from 12/9/19 to 7/9/21. The equity curves are plotted from Equity and Net Equity columns in Table 1. Plotted on the equity curves is the 2<sup>nd</sup> Order Polynomial curve. The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity. The brown line is the equity curve with commissions and the green dots are the new highs in net equity. The grey line is the DIA Daily Closing prices superimposed on the Equity Chart.

**Figure 2** presents a plot of the FixmVn Strategy buy/sells and the FixmVn Indicator on the DIA 5min bars for 6/23/2021 - 7/9/2021.

**Table 1** below presents a table of the 399 in-sample and out-of-sample windows, the **Filter** selected in-sample strategy inputs and the daily out-of-sample profit/loss results using the filter described above.

## Discussion of Strategy Performance

In Figure 3, Row 4 of the spreadsheet filter output are some statistics that are of interest for our filter. An interesting statistic is **Blw**. **Blw** is the maximum number of days the OSNP equity curve failed to make a new high. **Blw** is **45** days for this filter. This means that 45 trading days was the longest time that the equity for this strategy failed to make a new equity high. **%Wtr** is the percentage of all OOS trades that were wins or positive. For this filter, the **%Wtr=56%**. **%P** is the % winning oos days, **%P=65%**. The average oos winning trade to the average oos losing trade ratio(**oW|oL**) was **1.5**. **wpr=8** is the maximum number of consecutive winning oos periods(days) in a row and **lpr=4** is the maximum number of consecutive losing oos periods(days) in a row. The Largest losing trade in the whole period was **(\$692)** and the largest losing day was **(\$692)**.

In Figure 1, which presents a graph of the equity curve using the filter on the 399 trading days of out-of-sample data, notice how the equity curve follows the 2<sup>nd</sup> order polynomial trend line with an  $R^2$  of 0.984. The  $R^2$  only dropped to 0.982 for the net equity curve.

Using this filter, the strategy was able to generate \$11309 net equity after commissions of \$0 (many brokers today, 8/1/21, don't charge commissions) and slippage of \$4 trading 100 DIA ETF shares for 399 days. This period of time from 2/20/20 to 4/30/20 was a volatile down then up market. Yet the FixmVn strategy was able to adapt quite well.

In observing Table 1 we can see that this strategy and filter made trades from a low of no trades/day to a high of 7 trades/day with an average of 2.0 trades/day on the days it traded. For the no trade days, the strategy **input|filter** in the in-sample section didn't satisfy the metric filter and no trades were made the next trading day. The **input|filter** traded 151 days out of the 399 days or about 38% of the time.

## References

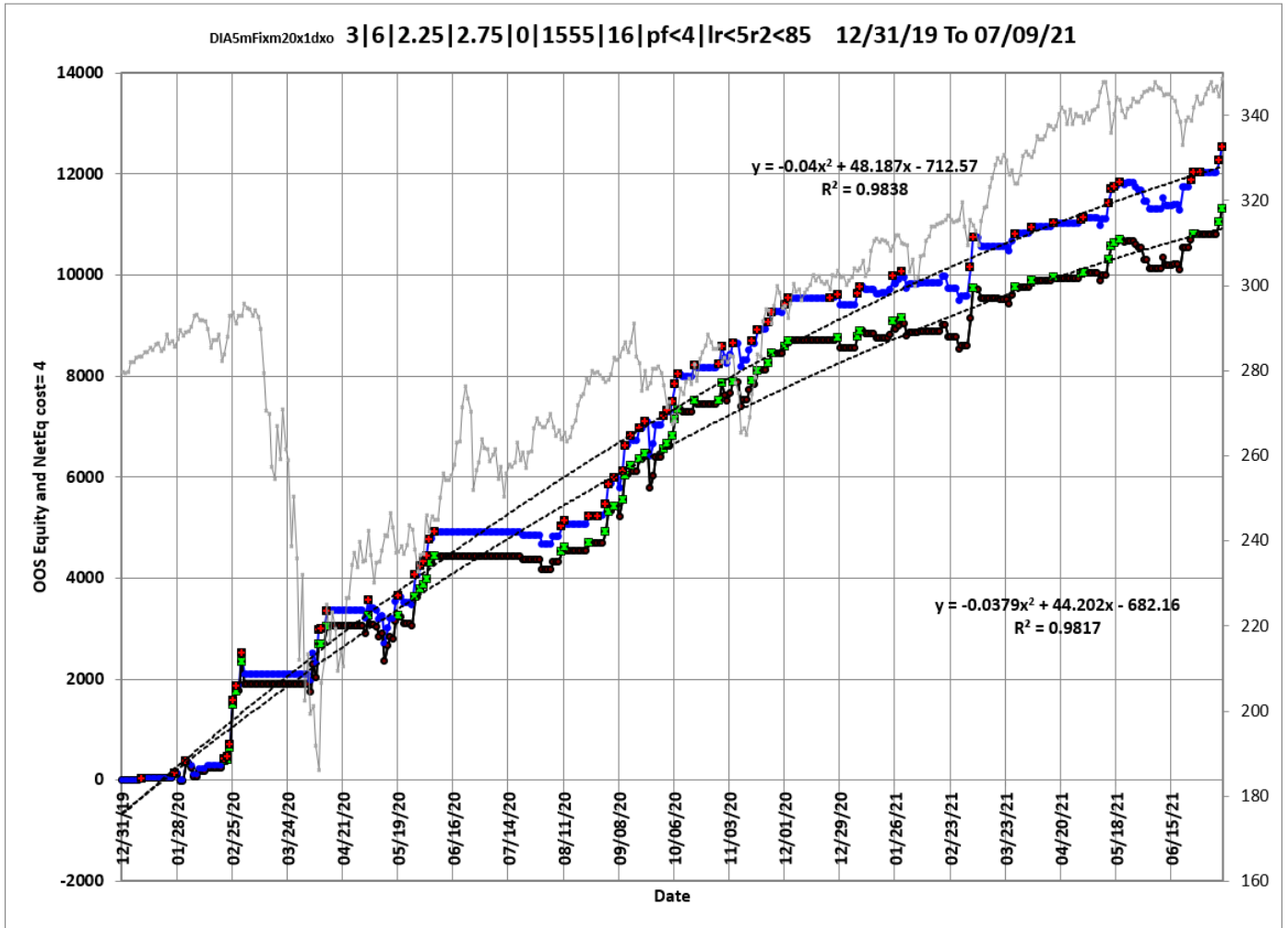
1. Efron, B., Tibshirani, R.J., (1993), "An Introduction to the Bootstrap", New York, Chapman & Hall/CRC.
2. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.

**Figure 1 Graph of FixmVn Strategy Equity Applying the Walk Forward Filter Each Day on the in-sample section on DIA 5min Bar Prices 12/9/2019 to 7/9/2020**

Note: The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity.

The brown line is the equity curve with commissions and the green dots are the new highs in net equity

The grey line is the DIA Daily Closing prices superimposed on the Equity Chart.



**Figure 2 Walk Forward Out-Of-Sample Performance Summary for nth Order Fixed Memory Polynomial Velocity Strategy DIA 5-minute bar chart from 6/25/21 to 7/9/21**





**Figure 3 Partial output of the Walk Forward Strategy Inputs with Metric Filters (WFINP)  
DIA ETF 5 min bars Using The FixmVn Strategy**

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	
1	DIA5mFixm20x1dxo	s12/31/19	e07/09/21	#399	AnyTnp						ISnt2			a(12.5)	s12.5	f444527					c=\$4							
2	pw N vup vdn xop xt mult <PF<LR-R2	toGP	toNP	aoGP	aoTr	ao#T	std	skew	kur	t	oW oL	%Wtr	%P	LLtr	LLp	eqDD	wpr	lpr	#	V20	Dev^2	KTau	eqR2	Blw	BE	tkr bl	Prob	
3	3 6 2.5 3 0 1555 16 pf<2 r2<70	13269	11389	85	28.2	3	363	-0.515	6.1	2.92	1.33	51	65	-706	-1437	-3768	11	4	156	19	1114	94	92	52	196	248	3.49E-12	
4	3 6 2.25 2.75 0 1555 16 pf<4 lr<5 r2<85	12537	11309	83	40.8	2	226	0.186	4.59	4.52	1.5	56	65	-692	-692	-985	8	4	151	17	675	95	97	45	82	1119	1.20E-12	
5	3 6 2.25 2.75 0 1555 16 pf<4 lr<5 r2<90	12618	11194	76	35.4	2.1	232	-0.107	4.93	4.2	1.45	55	65	-692	-766	-1166	8	4	167	17	694	94	97	61	95	658	8.92E-11	
6	3 6 2.75 3 0 1555 16 pf<2	12741	10961	81	28.6	2.8	328	-0.613	7.44	3.09	1.43	50	62	-706	-1437	-2762	13	3	158	11	1322	93	89	52	175	280	2.57E-11	
7	3 6 2.25 2.75 0 1555 16 pf<4 lr<5 r2<80	12071	10883	84	40.6	2.1	228	0.172	4.59	4.41	1.52	56	65	-692	-692	-985	8	4	144	17	656	94	97	45	86	1037	8.09E-13	
8	3 6 2.75 3 0 1555 16 pf<2 r2<90	12584	10828	81	28.7	2.8	331	-0.612	7.32	3.06	1.44	50	62	-706	-1437	-2762	13	3	155	11	1286	93	89	52	179	271	1.98E-11	
9	3 6 2.25 2.75 0 1555 16 pf<4 lr<5	12160	10664	69	32.5	2.1	231	-0.069	4.86	3.98	1.38	55	65	-729	-766	-1342	8	4	175	17	676	93	96	66	106	511	1.93E-09	
10	3 6 2.75 3 0 1555 16 pf<5 r2<75	12300	10404	74	25.9	2.9	321	-0.642	7.49	2.98	1.35	50	60	-706	-1437	-2762	13	4	166	11	1380	92	87	51	189	246	8.21E-10	
11	3 6 2.75 3 0 1555 16 r2<75	12300	10404	74	25.9	2.9	321	-0.642	7.49	2.98	1.35	50	60	-706	-1437	-2762	13	4	166	11	1380	92	87	51	189	246	8.21E-10	
12	3 6 2.75 3 0 1555 16 pf<6 r2<75	12300	10404	74	25.9	2.9	321	-0.642	7.49	2.98	1.35	50	60	-706	-1437	-2762	13	4	166	11	1380	92	87	51	189	246	8.21E-10	
13	3 6 2.75 3 0 1555 16 pf<4 r2<75	12208	10332	74	26	2.9	323	-0.642	7.42	2.96	1.36	50	60	-706	-1437	-2762	13	4	164	11	1357	92	87	51	192	241	6.98E-10	
14	3 6 2.75 2.75 0 1555 16 pf<6 lr<5 r2<75	11366	10266	82	41.3	2	245	0.912	6.31	3.93	1.76	52	63	-538	-489	-1393	11	4	139	20	808	92	95	44	109	718	2.16E-12	
15	3 6 2.75 2.75 0 1555 16 pf<5 lr<5 r2<75	11366	10266	82	41.3	2	245	0.912	6.31	3.93	1.76	52	63	-538	-489	-1393	11	4	139	20	808	92	95	44	109	718	2.16E-12	
16	3 6 2.75 3 0 1555 16 pf<5 r2<80	12226	10246	69	24.7	2.8	316	-0.606	7.38	2.9	1.33	51	60	-706	-1437	-2762	13	4	178	7	1556	88	83	78	200	137	9.55E-09	

**The WFINP Filter Output Columns are defined as follows: OOS=out-of-sample**

**Row 1** DIA5Fixm20x1dxo is the PWFO output files abbreviation, First OOS Day End Date (12/09/19), Last OOS Day End Date (07/09/21), **Number of days(#399)** a=average of bootstrap random picks. s= standard deviation of bootstrap random picks. f=number of different filters examined. c= slippage and round-trip trade cost(c=\$4).

**The WFINP AVE File Output Cols are defined as follows**

- **Row 2 to Last Row Columns: A through AA**

**Col A: The Strategy Input/Filter Names**

Row 4: 3|6|2.25|2.75|0|1555|16|pf<4|lr<5|r2<85: The inputs 3|6|2.25|2.75|0|1555|16 for all in-sample files that have PF≤4 and/or lr≤5 and/or R2 ≤85.

**Col B: toGP** Total out-of-sample(oos) gross profit for these 399 oos periods (for this run periods = weeks).

**Col C: toNP** Total out-of-sample(oos) Net profit (toGP-Number Of Trade Weeks\*cost) for the 399 oos periods.

**Col D: aoGP** Average oos gross profit for the # oos periods

**Col E: aoTr** Average oos profit per trade

**Col F: ao#T** Average number of oos trades per week

**Col G: std** The standard deviation of the # oos period profits and losses

**Col H: skew** The Skew statistic of the # oos period profits and losses

**Col I: kur** The kurtosis statistic of the # oos period profits and losses

**Col J: t** The student t statistic for the # oos periods. The higher the t statistic the higher the probability that this result was not due to pure chance

**Col K: oW|oL** Ratio of average oos winning trades divided by average oos losing trades.

**Col L: %Wtr** The percentage if oos winning trades

**Col M: %P** percent of all oos periods that were profitable.

**Col N: LLtr** The largest losing oos trade in all oos periods

**Col O: LLp** The largest losing oos period

**Col P: eqDD** The oos equity drawdown

**Col Q: wpr** The largest number of winning oos periods (weeks) in a row.

**Col R: lpr** The largest number of losing oos periods in a row

**Col S: #** The number of oos periods this filter produced any profit or loss. Note for some oos periods there can be no strategy inputs that satisfy a given filters criteria, and no trades will be made during that period.

**Col T: v20** The straight-line trend of the oos equity curve for the last 20 bars.

**Col U: Dev^2** A measure of equity curve smoothness. The square root of the average (equity curve minus a straight line)^2)

**Col V: *KTau*** The Kendall rank coefficient is often used as a test statistic in a statistical hypothesis test to establish whether two variables may be regarded as statistically dependent. This test is non-parametric, as it does not rely on any assumptions on the distributions of X or Y or the distribution of (X,Y)

**Col W: *eqR2*** The correlation coefficient( $R^2$ ) of a straight line fit to the equity curve.

**Col X: *Blw*** The maximum number of oos periods the oos equity curve failed to make a new high.

**Col Y: *BE*** Break even in oos periods. Assuming the average and standard deviation are from a normal distribution, this is the number of oos periods you would have to trade to have a 98% probability that your oos equity is above zero.

**Col Z: *tkr/bl***  $=100 * t * K\tau * eqR2 / Blw / BE$ . This is measure of the best equity curve.

**Col AA: *Prob*** The probability that the filters oos toNP was due to pure chance. Row 1 lists the random bootstrap average for the 399 out-of-sample files of (\$12.5) with a bootstrap standard deviation of \$12.5. (Note. The average for the random selection is computed as the Average Random toNP/399) The average net weekly for the filter would be the filter toNP/ (# of OOS) periods traded would be the filter toNP/# of OOS periods traded or  $11309/151=74.9$ . The probability of obtaining our filters average daily net profit of **74.9** is  $1.20 \times 10^{-12}$  which is **7.0** standard deviations from the bootstrap average. For our filter, in row 4 above, the expected number of cases that we could obtain by pure chance that would match or exceed **\$74.9** is  $[1 - (1.20 \times 10^{-12})^{444527}] \sim 444527 * 1.20 \times 10^{-12} = 0.00000533$  where **444527** is the total number of different filters we looked at in this run. This number is much much less than one, so it is improbable that our result was due to pure chance.

**Table 1 Walk Forward Out-Of-Sample Performance Summary for the DIA 5-min FixmVn Strategy**

DIA-5 min bars 12/9/2019 - 7/9/2021.

Filter: 3|6|2.25|2.75|0|1555|16|pf<4|lr<5|r2<85: The inputs 3|6|2.25|2.75|0|1555|16 for all in-sample files that have PF≤4 and/or lr≤5 and/or R2 ≤85. are used to trade in the following out-of-sample sections.

IS-pf = In-sample pf

IS-lr = in-sample losers in a row(lr)

IS-r2 = in-sample equity r2

osnp = Daily out-of-sample gross profit in \$

NOnp\$4 = Daily out-of-sample net profit in \$ = osnp-ont\*4.

ont = The number of trades in the out-of-sample day

ownp = winning profits in the out-of-sample day.

ownt = number of winning trades in the out-of-sample day

ollt = The largest losing trade in the out-of-sample day in \$.

odd = The drawdown in the out-of-sample day in \$.

EQ=Equity = Running Sum of daily out-of-sample gross profits \$

NetEq=Net Equity = running sum of the daily out-of-sample net profits in \$

Note: Blank rows indicate that no out-of-sample trades were made that day

Date	IS pf	IS lr	IS r2	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
12/31/2019	7.38	1	87	0	0	0	0	0	0	0	0	0
1/1/2020	4.79	1	38	0	0	0	0	0	0	0	0	0
1/2/2020	4.79	1	38	0	0	0	0	0	0	0	0	0
1/3/2020	8.93	1	75	0	0	0	0	0	0	0	0	0
1/6/2020	3.13	2	46	0	0	0	0	0	0	0	0	0
1/7/2020	3.13	2	46	0	0	0	0	0	0	0	0	0
1/8/2020	3.13	2	46	0	0	0	0	0	0	0	0	0
1/9/2020	3.13	2	46	39	35	1	39	1	0	0	39	35
1/10/2020	2.34	2	37	0	0	0	0	0	0	0	39	35
1/13/2020	2.34	2	37	0	0	0	0	0	0	0	39	35
1/14/2020	4.35	2	-17	0	0	0	0	0	0	0	39	35
1/15/2020	4.35	2	-17	0	0	0	0	0	0	0	39	35
1/16/2020	4.35	2	-17	0	0	0	0	0	0	0	39	35
1/17/2020	6.16	2	28	0	0	0	0	0	0	0	39	35
1/20/2020	6.16	2	28	0	0	0	0	0	0	0	39	35
1/21/2020	6.16	2	28	0	0	0	0	0	0	0	39	35
1/22/2020	6.16	2	28	0	0	0	0	0	0	0	39	35
1/23/2020	6.16	2	28	0	0	0	0	0	0	0	39	35
1/24/2020	1.61	2	0	0	0	0	0	0	0	0	39	35
1/27/2020	1.61	2	0	88	76	3	102	2	-14	-14	127	111
1/28/2020	2.01	2	0	0	0	0	0	0	0	0	127	111
1/29/2020	2.01	2	0	(142)	(146)	1	0	0	-142	-142	(15)	(35)
1/30/2020	1.18	2	-11	0	0	0	0	0	0	0	(15)	(35)
1/31/2020	0.67	2	-9	384	380	1	384	1	0	0	369	345
2/3/2020	2.09	2	10	(19)	(23)	1	0	0	-19	-19	350	322
2/4/2020	1.96	2	28	(65)	(77)	3	4	1	-45	-69	285	245
2/5/2020	1.62	2	48	(180)	(188)	2	7	1	-187	-187	105	57
2/6/2020	1.10	2	30	0	0	0	0	0	0	0	105	57
2/7/2020	1.03	2	27	107	103	1	107	1	0	0	212	160
2/10/2020	1.22	2	23	0	0	0	0	0	0	0	212	160
2/11/2020	1.22	2	23	0	0	0	0	0	0	0	212	160

Date	IS pf	IS lr	IS r2	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
2/12/2020	1.22	2	23	71	67	1	71	1	0	0	283	227
2/13/2020	1.34	2	26	0	0	0	0	0	0	0	283	227
2/14/2020	1.19	2	29	0	0	0	0	0	0	0	283	227
2/17/2020	1.19	2	29	0	0	0	0	0	0	0	283	227
2/18/2020	1.19	2	29	0	0	0	0	0	0	0	283	227
2/19/2020	1.19	2	29	0	0	0	0	0	0	0	283	227
2/20/2020	1.19	2	29	147	143	1	147	1	0	0	430	370
2/21/2020	1.91	2	31	30	22	2	59	1	-29	-29	460	392
2/24/2020	1.92	2	48	252	240	3	299	2	-47	-47	712	632
2/25/2020	2.19	2	45	870	862	2	937	1	-67	-67	1582	1494
2/26/2020	3.60	2	47	286	266	5	457	3	-121	-171	1868	1760
2/27/2020	4.20	2	71	0	0	0	0	0	0	0	1868	1760
2/29/2020	1.98	2	75	640	580	15	1348	11	-331	-374	2508	2340
3/2/2020	1.78	2	62	(416)	(448)	8	569	1	-446	-985	2092	1892
3/3/2020	1.41	7	43	0	0	0	0	0	0	0	2092	1892
3/4/2020	0.73	9	-21	0	0	0	0	0	0	0	2092	1892
3/5/2020	0.87	9	-34	0	0	0	0	0	0	0	2092	1892
3/6/2020	0.76	9	-45	0	0	0	0	0	0	0	2092	1892
3/9/2020	0.68	9	-59	0	0	0	0	0	0	0	2092	1892
3/10/2020	0.70	9	-71	0	0	0	0	0	0	0	2092	1892
3/11/2020	0.66	9	-81	0	0	0	0	0	0	0	2092	1892
3/12/2020	0.66	9	-85	0	0	0	0	0	0	0	2092	1892
3/13/2020	0.76	9	-83	0	0	0	0	0	0	0	2092	1892
3/16/2020	0.83	9	-77	0	0	0	0	0	0	0	2092	1892
3/17/2020	0.76	9	-78	0	0	0	0	0	0	0	2092	1892
3/18/2020	0.82	9	-76	0	0	0	0	0	0	0	2092	1892
3/19/2020	0.82	9	-73	0	0	0	0	0	0	0	2092	1892
3/20/2020	0.87	9	-65	0	0	0	0	0	0	0	2092	1892
3/23/2020	0.87	9	-60	0	0	0	0	0	0	0	2092	1892
3/24/2020	0.87	9	-53	0	0	0	0	0	0	0	2092	1892
3/25/2020	0.83	9	-51	0	0	0	0	0	0	0	2092	1892
3/26/2020	0.86	9	-43	0	0	0	0	0	0	0	2092	1892
3/27/2020	0.87	9	-26	0	0	0	0	0	0	0	2092	1892
3/30/2020	0.89	9	-10	0	0	0	0	0	0	0	2092	1892
3/31/2020	0.92	9	-2	0	0	0	0	0	0	0	2092	1892
4/1/2020	1.07	8	14	0	0	0	0	0	0	0	2092	1892
4/2/2020	1.01	8	22	0	0	0	0	0	0	0	2092	1892
4/3/2020	1.07	5	39	(140)	(152)	3	68	1	-191	-191	1952	1740
4/6/2020	1.10	5	52	549	537	3	710	2	-161	-161	2501	2277
4/7/2020	1.14	5	61	(201)	(245)	11	535	5	-208	-467	2300	2032
4/8/2020	1.19	5	67	673	661	3	673	3	0	0	2973	2693
4/9/2020	1.23	5	67	15	3	3	176	1	-154	-161	2988	2696
4/10/2020	1.20	5	73	0	0	0	0	0	0	0	2988	2696
4/13/2020	1.19	5	85	361	353	2	361	2	0	0	3349	3049
4/14/2020	1.38	5	86	0	0	0	0	0	0	0	3349	3049
4/15/2020	1.40	5	86	0	0	0	0	0	0	0	3349	3049
4/16/2020	1.50	4	86	0	0	0	0	0	0	0	3349	3049
4/17/2020	1.46	4	88	0	0	0	0	0	0	0	3349	3049
4/20/2020	1.61	4	91	0	0	0	0	0	0	0	3349	3049
4/21/2020	1.74	3	90	0	0	0	0	0	0	0	3349	3049
4/22/2020	1.89	3	90	0	0	0	0	0	0	0	3349	3049
4/23/2020	1.96	3	88	0	0	0	0	0	0	0	3349	3049
4/24/2020	2.23	3	86	0	0	0	0	0	0	0	3349	3049
4/27/2020	1.94	4	86	0	0	0	0	0	0	0	3349	3049
4/28/2020	1.64	4	86	0	0	0	0	0	0	0	3349	3049
4/29/2020	1.49	4	90	0	0	0	0	0	0	0	3349	3049
4/30/2020	1.75	4	88	0	0	0	0	0	0	0	3349	3049
5/1/2020	1.75	4	84	(137)	(145)	2	111	1	-248	-248	3212	2904

Date	IS pf	IS lr	IS r2	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
5/4/2020	1.74	4	80	352	344	2	352	2	0	0	3564	3248
5/5/2020	1.71	4	75	(155)	(167)	3	15	1	-131	-170	3409	3081
5/6/2020	1.96	4	55	0	0	0	0	0	0	0	3409	3081
5/7/2020	1.58	4	48	(59)	(63)	1	0	0	-59	-59	3350	3018
5/8/2020	1.57	4	37	(171)	(179)	2	2	1	-173	-173	3179	2839
5/11/2020	1.43	4	32	61	53	2	116	1	-55	-55	3240	2892
5/12/2020	1.26	4	20	(533)	(537)	1	0	0	-533	-533	2707	2355
5/13/2020	0.96	4	6	301	297	1	301	1	0	0	3008	2652
5/14/2020	1.12	4	0	194	178	4	556	2	-248	-362	3202	2830
5/15/2020	1.10	4	-8	(23)	(31)	2	120	1	-143	-143	3179	2799
5/18/2020	1.03	4	-26	350	338	3	350	3	0	0	3529	3137
5/19/2020	1.12	4	-42	130	118	3	216	2	-86	-86	3659	3255
5/20/2020	1.03	4	-35	(31)	(39)	2	0	0	-31	-31	3628	3216
5/21/2020	0.91	4	-18	(117)	(121)	1	0	0	-117	-117	3511	3095
5/22/2020	0.92	3	-7	0	0	0	0	0	0	0	3511	3095
5/25/2020	0.87	3	-1	0	0	0	0	0	0	0	3511	3095
5/26/2020	1.00	3	0	(36)	(48)	3	58	2	-94	-94	3475	3047
5/27/2020	1.13	3	6	586	574	3	586	3	0	0	4061	3621
5/28/2020	1.38	3	14	0	0	0	0	0	0	0	4061	3621
5/29/2020	1.34	3	17	169	157	3	223	1	-34	-54	4230	3778
6/1/2020	1.54	3	37	107	99	2	168	1	-61	-61	4337	3877
6/2/2020	1.40	3	52	99	95	1	99	1	0	0	4436	3972
6/3/2020	1.58	3	73	333	329	1	333	1	0	0	4769	4301
6/4/2020	1.77	3	74	0	0	0	0	0	0	0	4769	4301
6/5/2020	1.83	3	78	141	129	3	141	3	0	0	4910	4430
6/8/2020	2.13	3	87	0	0	0	0	0	0	0	4910	4430
6/9/2020	2.27	3	91	0	0	0	0	0	0	0	4910	4430
6/10/2020	3.03	3	92	0	0	0	0	0	0	0	4910	4430
6/11/2020	2.83	3	93	0	0	0	0	0	0	0	4910	4430
6/12/2020	2.08	3	88	0	0	0	0	0	0	0	4910	4430
6/15/2020	1.34	6	48	0	0	0	0	0	0	0	4910	4430
6/16/2020	0.93	6	11	0	0	0	0	0	0	0	4910	4430
6/17/2020	0.80	6	-2	0	0	0	0	0	0	0	4910	4430
6/18/2020	0.73	6	-18	0	0	0	0	0	0	0	4910	4430
6/19/2020	0.72	6	-35	0	0	0	0	0	0	0	4910	4430
6/22/2020	0.64	6	-48	0	0	0	0	0	0	0	4910	4430
6/23/2020	0.69	6	-51	0	0	0	0	0	0	0	4910	4430
6/24/2020	0.70	6	-66	0	0	0	0	0	0	0	4910	4430
6/25/2020	0.63	6	-80	0	0	0	0	0	0	0	4910	4430
6/26/2020	0.72	6	-80	0	0	0	0	0	0	0	4910	4430
6/29/2020	0.79	6	-82	0	0	0	0	0	0	0	4910	4430
6/30/2020	0.79	6	-79	0	0	0	0	0	0	0	4910	4430
7/1/2020	0.77	6	-79	0	0	0	0	0	0	0	4910	4430
7/2/2020	0.70	6	-80	0	0	0	0	0	0	0	4910	4430
7/3/2020	0.65	6	-73	0	0	0	0	0	0	0	4910	4430
7/6/2020	0.62	6	-71	0	0	0	0	0	0	0	4910	4430
7/7/2020	0.58	6	-66	0	0	0	0	0	0	0	4910	4430
7/8/2020	0.59	6	-59	0	0	0	0	0	0	0	4910	4430
7/9/2020	0.59	6	-56	0	0	0	0	0	0	0	4910	4430
7/10/2020	0.57	6	-43	0	0	0	0	0	0	0	4910	4430
7/13/2020	0.58	6	-5	0	0	0	0	0	0	0	4910	4430
7/14/2020	0.64	6	0	0	0	0	0	0	0	0	4910	4430
7/15/2020	0.87	6	8	0	0	0	0	0	0	0	4910	4430
7/16/2020	0.94	6	11	0	0	0	0	0	0	0	4910	4430
7/17/2020	1.01	5	12	0	0	0	0	0	0	0	4910	4430
7/20/2020	1.29	5	0	0	0	0	0	0	0	0	4910	4430
7/21/2020	1.16	5	0	(60)	(64)	1	0	0	-60	-60	4850	4366
7/22/2020	1.11	5	-8	0	0	0	0	0	0	0	4850	4366

Date	IS pf	IS lr	IS r2	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
7/23/2020	1.02	5	-70	0	0	0	0	0	0	0	4850	4366
7/24/2020	0.80	5	-81	0	0	0	0	0	0	0	4850	4366
7/27/2020	0.54	5	-79	0	0	0	0	0	0	0	4850	4366
7/28/2020	0.43	5	-75	0	0	0	0	0	0	0	4850	4366
7/29/2020	0.43	5	-75	0	0	0	0	0	0	0	4850	4366
7/30/2020	0.43	5	-75	(189)	(201)	3	104	1	-231	-293	4661	4165
7/31/2020	0.53	3	-67	0	0	0	0	0	0	0	4661	4165
8/3/2020	0.53	3	-67	0	0	0	0	0	0	0	4661	4165
8/4/2020	0.53	3	-55	0	0	0	0	0	0	0	4661	4165
8/5/2020	0.58	3	-41	163	159	1	163	1	0	0	4824	4324
8/6/2020	0.64	3	-33	0	0	0	0	0	0	0	4824	4324
8/7/2020	0.66	3	-9	0	0	0	0	0	0	0	4824	4324
8/10/2020	1.02	2	-2	199	195	1	199	1	0	0	5023	4519
8/11/2020	1.94	2	-27	102	90	3	303	2	-201	-201	5125	4609
8/12/2020	1.11	2	13	(66)	(74)	2	0	0	-49	-66	5059	4535
8/13/2020	1.24	3	43	0	0	0	0	0	0	0	5059	4535
8/14/2020	1.24	3	43	0	0	0	0	0	0	0	5059	4535
8/17/2020	1.24	3	43	0	0	0	0	0	0	0	5059	4535
8/18/2020	1.24	3	43	0	0	0	0	0	0	0	5059	4535
8/19/2020	1.37	3	40	0	0	0	0	0	0	0	5059	4535
8/20/2020	1.37	3	40	0	0	0	0	0	0	0	5059	4535
8/21/2020	1.37	3	40	0	0	0	0	0	0	0	5059	4535
8/24/2020	1.37	3	40	163	159	1	163	1	0	0	5222	4694
8/25/2020	1.66	3	48	0	0	0	0	0	0	0	5222	4694
8/26/2020	1.66	3	48	0	0	0	0	0	0	0	5222	4694
8/27/2020	1.66	3	48	3	(1)	1	3	1	0	0	5225	4693
8/28/2020	3.11	3	40	0	0	0	0	0	0	0	5225	4693
8/31/2020	3.11	3	40	0	0	0	0	0	0	0	5225	4693
9/1/2020	3.11	3	40	228	224	1	228	1	0	0	5453	4917
9/2/2020	3.97	3	56	402	398	1	402	1	0	0	5855	5315
9/3/2020	4.86	3	53	0	0	0	0	0	0	0	5855	5315
9/4/2020	1.15	3	8	129	109	5	700	2	-262	-309	5984	5424
9/7/2020	1.18	4	-6	0	0	0	0	0	0	0	5984	5424
9/8/2020	1.06	4	-10	(201)	(217)	4	123	1	-139	-324	5783	5207
9/9/2020	0.89	4	-11	341	337	1	341	1	0	0	6124	5544
9/10/2020	1.12	4	-13	502	494	2	525	1	-23	-23	6626	6038
9/11/2020	1.40	4	0	0	0	0	0	0	0	0	6626	6038
9/14/2020	1.40	4	0	193	189	1	193	1	0	0	6819	6227
9/15/2020	1.51	4	3	(110)	(122)	3	45	1	-134	-134	6709	6105
9/16/2020	1.41	4	22	0	0	0	0	0	0	0	6709	6105
9/17/2020	1.41	4	22	250	242	2	250	2	0	0	6959	6347
9/18/2020	1.54	4	34	0	0	0	0	0	0	0	6959	6347
9/21/2020	1.54	4	34	137	125	3	137	3	0	0	7096	6472
9/22/2020	1.53	4	51	0	0	0	0	0	0	0	7096	6472
9/23/2020	1.53	4	51	(692)	(696)	1	0	0	-692	-692	6404	5776
9/24/2020	1.12	4	47	239	235	1	239	1	0	0	6643	6011
9/25/2020	1.21	4	48	375	371	1	375	1	0	0	7018	6382
9/28/2020	1.36	4	51	14	2	3	112	2	-98	-98	7032	6384
9/29/2020	1.35	4	57	0	0	0	0	0	0	0	7032	6384
9/30/2020	1.26	4	61	167	163	1	167	1	0	0	7199	6547
10/1/2020	1.18	4	76	126	118	2	175	1	-49	-49	7325	6665
10/2/2020	1.77	3	80	(42)	(58)	4	98	1	-70	-140	7283	6607
10/5/2020	1.88	3	73	218	214	1	218	1	0	0	7501	6821
10/6/2020	2.02	3	75	336	328	2	371	1	-35	-35	7837	7149
10/7/2020	2.72	3	71	194	182	3	233	2	-39	-39	8031	7331
10/8/2020	2.55	3	76	0	0	0	0	0	0	0	8031	7331
10/9/2020	2.16	3	73	(38)	(42)	1	0	0	-38	-38	7993	7289
10/12/2020	2.10	3	76	0	0	0	0	0	0	0	7993	7289

Date	IS pf	IS lr	IS r2	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
10/13/2020	1.94	3	76	0	0	0	0	0	0	0	7993	7289
10/14/2020	2.18	3	76	0	0	0	0	0	0	0	7993	7289
10/15/2020	2.18	3	76	227	219	2	232	1	-5	-5	8220	7508
10/16/2020	2.15	3	81	(62)	(66)	1	0	0	-62	-62	8158	7442
10/19/2020	2.04	3	83	0	0	0	0	0	0	0	8158	7442
10/20/2020	1.92	3	93	0	0	0	0	0	0	0	8158	7442
10/21/2020	1.96	3	94	0	0	0	0	0	0	0	8158	7442
10/22/2020	3.58	3	92	0	0	0	0	0	0	0	8158	7442
10/23/2020	3.20	3	91	0	0	0	0	0	0	0	8158	7442
10/26/2020	2.61	3	91	0	0	0	0	0	0	0	8158	7442
10/27/2020	1.87	3	76	0	0	0	0	0	0	0	8158	7442
10/28/2020	1.87	3	76	84	72	3	326	1	-146	-242	8242	7514
10/29/2020	1.60	3	44	345	341	1	345	1	0	0	8587	7855
10/30/2020	1.84	3	37	(235)	(239)	1	0	0	-235	-235	8352	7616
11/2/2020	1.60	3	3	(114)	(122)	2	16	1	-130	-130	8238	7494
11/3/2020	1.28	3	-1	182	170	3	284	2	-102	-102	8420	7664
11/4/2020	1.16	3	-5	232	228	1	232	1	0	0	8652	7892
11/5/2020	1.19	3	-3	(11)	(23)	3	75	1	-74	-86	8641	7869
11/6/2020	1.17	3	2	0	0	0	0	0	0	0	8641	7869
11/9/2020	1.20	3	2	(457)	(469)	3	48	1	-296	-457	8184	7400
11/10/2020	0.90	3	0	128	124	1	128	1	0	0	8312	7524
11/11/2020	0.97	3	0	0	0	0	0	0	0	0	8312	7524
11/12/2020	0.97	3	0	198	194	1	198	1	0	0	8510	7718
11/13/2020	0.96	3	2	183	179	1	183	1	0	0	8693	7897
11/16/2020	1.09	3	9	(54)	(62)	2	2	1	-56	-56	8639	7835
11/17/2020	1.06	3	17	280	272	2	280	2	0	0	8919	8107
11/18/2020	1.18	3	37	0	0	0	0	0	0	0	8919	8107
11/19/2020	1.30	3	45	0	0	0	0	0	0	0	8919	8107
11/20/2020	1.30	3	45	0	0	0	0	0	0	0	8919	8107
11/23/2020	1.30	3	45	146	142	1	146	1	0	0	9065	8249
11/24/2020	1.67	2	57	199	195	1	199	1	0	0	9264	8444
11/25/2020	1.82	2	61	0	0	0	0	0	0	0	9264	8444
11/26/2020	1.92	2	49	0	0	0	0	0	0	0	9264	8444
11/27/2020	1.61	2	58	0	0	0	0	0	0	0	9264	8444
11/30/2020	2.04	2	58	(6)	(14)	2	43	1	-49	-49	9258	8430
12/1/2020	2.28	2	63	153	145	2	153	2	0	0	9411	8575
12/2/2020	2.42	2	67	131	123	2	180	1	-49	-49	9542	8698
12/3/2020	2.19	2	79	0	0	0	0	0	0	0	9542	8698
12/4/2020	2.37	1	92	0	0	0	0	0	0	0	9542	8698
12/7/2020	2.55	1	93	0	0	0	0	0	0	0	9542	8698
12/8/2020	10.62	1	95	0	0	0	0	0	0	0	9542	8698
12/9/2020	9.79	1	94	0	0	0	0	0	0	0	9542	8698
12/10/2020	9.79	1	94	0	0	0	0	0	0	0	9542	8698
12/11/2020	5.81	1	93	0	0	0	0	0	0	0	9542	8698
12/14/2020	5.05	1	92	0	0	0	0	0	0	0	9542	8698
12/15/2020	8.70	1	90	0	0	0	0	0	0	0	9542	8698
12/16/2020	3.17	2	89	0	0	0	0	0	0	0	9542	8698
12/17/2020	3.17	2	89	0	0	0	0	0	0	0	9542	8698
12/18/2020	3.17	2	87	0	0	0	0	0	0	0	9542	8698
12/21/2020	3.17	2	87	0	0	0	0	0	0	0	9542	8698
12/22/2020	3.09	2	84	0	0	0	0	0	0	0	9542	8698
12/23/2020	2.63	2	81	3	(1)	1	3	1	0	0	9545	8697
12/24/2020	2.64	2	83	0	0	0	0	0	0	0	9545	8697
12/25/2020	2.64	2	83	0	0	0	0	0	0	0	9545	8697
12/28/2020	2.64	2	83	65	61	1	65	1	0	0	9610	8758
12/29/2020	3.03	2	79	(206)	(210)	1	0	0	-206	-206	9404	8548
12/30/2020	1.72	2	60	0	0	0	0	0	0	0	9404	8548
12/31/2020	1.54	2	40	0	0	0	0	0	0	0	9404	8548

Date	IS pf	IS lr	IS r2	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
1/1/2021	1.54	2	40	0	0	0	0	0	0	0	9404	8548
1/4/2021	1.31	2	33	0	0	0	0	0	0	0	9404	8548
1/5/2021	1.31	2	33	0	0	0	0	0	0	0	9404	8548
1/6/2021	1.31	2	33	231	223	2	309	1	-78	-78	9635	8771
1/7/2021	1.65	2	49	130	126	1	130	1	0	0	9765	8897
1/8/2021	1.99	2	37	0	0	0	0	0	0	0	9765	8897
1/11/2021	1.99	2	37	(68)	(72)	1	0	0	-68	-68	9697	8825
1/12/2021	1.12	2	49	0	0	0	0	0	0	0	9697	8825
1/13/2021	1.83	1	66	0	0	0	0	0	0	0	9697	8825
1/14/2021	1.83	1	66	0	0	0	0	0	0	0	9697	8825
1/15/2021	1.82	1	58	(78)	(86)	2	0	0	-44	-78	9619	8739
1/18/2021	1.50	3	43	0	0	0	0	0	0	0	9619	8739
1/19/2021	1.18	3	21	11	3	2	31	1	-20	-20	9630	8742
1/20/2021	1.20	4	10	0	0	0	0	0	0	0	9630	8742
1/21/2021	1.19	4	4	0	0	0	0	0	0	0	9630	8742
1/22/2021	1.19	4	4	86	78	2	86	2	0	0	9716	8820
1/25/2021	1.38	4	8	260	256	1	260	1	0	0	9976	9076
1/26/2021	1.81	4	26	(150)	(154)	1	0	0	-150	-150	9826	8922
1/27/2021	2.07	4	18	82	74	2	169	1	-87	-87	9908	8996
1/28/2021	2.05	4	41	156	152	1	156	1	0	0	10064	9148
1/29/2021	2.37	4	52	(109)	(117)	2	36	1	-145	-145	9955	9031
2/1/2021	1.88	4	62	(234)	(242)	2	0	0	-152	-234	9721	8789
2/2/2021	1.37	4	43	86	74	3	153	1	-49	-67	9807	8863
2/3/2021	1.43	5	18	0	0	0	0	0	0	0	9807	8863
2/4/2021	1.20	5	13	0	0	0	0	0	0	0	9807	8863
2/5/2021	1.05	5	14	6	(2)	2	55	1	-49	-49	9813	8861
2/8/2021	1.05	5	10	19	15	1	19	1	0	0	9832	8876
2/9/2021	1.16	5	8	0	0	0	0	0	0	0	9832	8876
2/10/2021	1.16	5	8	0	0	0	0	0	0	0	9832	8876
2/11/2021	1.16	5	8	0	0	0	0	0	0	0	9832	8876
2/12/2021	1.16	5	8	0	0	0	0	0	0	0	9832	8876
2/15/2021	1.28	5	1	0	0	0	0	0	0	0	9832	8876
2/16/2021	1.28	5	1	0	0	0	0	0	0	0	9832	8876
2/17/2021	1.28	5	-3	0	0	0	0	0	0	0	9832	8876
2/18/2021	1.28	5	-3	144	136	2	144	2	0	0	9976	9012
2/19/2021	1.47	5	0	0	0	0	0	0	0	0	9976	9012
2/22/2021	1.36	5	-17	(246)	(250)	1	0	0	-246	-246	9730	8762
2/23/2021	0.75	5	-18	0	0	0	0	0	0	0	9730	8762
2/24/2021	0.88	5	-24	0	0	0	0	0	0	0	9730	8762
2/25/2021	0.76	5	-18	0	0	0	0	0	0	0	9730	8762
2/26/2021	0.55	5	-7	(234)	(242)	2	0	0	-186	-234	9496	8520
3/1/2021	0.45	4	-10	90	78	3	96	2	-6	-6	9586	8598
3/2/2021	0.78	3	-31	0	0	0	0	0	0	0	9586	8598
3/3/2021	0.59	3	-56	0	0	0	0	0	0	0	9586	8598
3/4/2021	0.59	3	-56	566	546	5	729	2	-115	-163	10152	9144
3/5/2021	1.49	3	-2	599	587	3	724	2	-125	-125	10751	9731
3/8/2021	2.21	3	22	0	0	0	0	0	0	0	10751	9731
3/9/2021	2.19	3	27	(29)	(33)	1	0	0	-29	-29	10722	9698
3/10/2021	2.11	3	39	(156)	(164)	2	36	1	-192	-192	10566	9534
3/11/2021	1.74	3	53	0	0	0	0	0	0	0	10566	9534
3/12/2021	1.74	3	53	0	0	0	0	0	0	0	10566	9534
3/15/2021	1.74	3	53	(9)	(13)	1	0	0	-9	-9	10557	9521
3/16/2021	1.72	3	57	0	0	0	0	0	0	0	10557	9521
3/17/2021	1.72	3	57	0	0	0	0	0	0	0	10557	9521
3/18/2021	1.72	3	57	0	0	0	0	0	0	0	10557	9521
3/19/2021	1.58	3	74	3	(1)	1	3	1	0	0	10560	9520
3/22/2021	1.58	3	75	0	0	0	0	0	0	0	10560	9520
3/23/2021	2.09	3	80	0	0	0	0	0	0	0	10560	9520



Date	IS pf	IS lr	IS r2	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
3/24/2021	2.09	3	80	(97)	(101)	1	0	0	-97	-97	10463	9419
3/25/2021	1.86	3	78	194	186	2	300	1	-106	-106	10657	9605
3/26/2021	1.96	3	76	153	149	1	153	1	0	0	10810	9754
3/29/2021	2.81	3	75	0	0	0	0	0	0	0	10810	9754
3/30/2021	2.70	3	66	0	0	0	0	0	0	0	10810	9754
3/31/2021	2.70	3	66	0	0	0	0	0	0	0	10810	9754
4/1/2021	2.70	3	66	0	0	0	0	0	0	0	10810	9754
4/2/2021	2.18	2	18	0	0	0	0	0	0	0	10810	9754
4/5/2021	1.14	2	-2	139	127	3	157	2	-18	-18	10949	9881
4/6/2021	1.44	2	20	0	0	0	0	0	0	0	10949	9881
4/7/2021	1.54	2	32	0	0	0	0	0	0	0	10949	9881
4/8/2021	2.67	2	66	0	0	0	0	0	0	0	10949	9881
4/9/2021	2.67	2	66	0	0	0	0	0	0	0	10949	9881
4/12/2021	2.67	2	66	0	0	0	0	0	0	0	10949	9881
4/13/2021	2.77	2	73	0	0	0	0	0	0	0	10949	9881
4/14/2021	2.77	2	73	0	0	0	0	0	0	0	10949	9881
4/15/2021	2.77	2	73	87	83	1	87	1	0	0	11036	9964
4/16/2021	3.17	2	81	(23)	(31)	2	1	1	-24	-24	11013	9933
4/19/2021	2.85	2	86	0	0	0	0	0	0	0	11013	9933
4/20/2021	2.85	2	86	0	0	0	0	0	0	0	11013	9933
4/21/2021	2.85	2	86	0	0	0	0	0	0	0	11013	9933
4/22/2021	4.72	1	81	0	0	0	0	0	0	0	11013	9933
4/23/2021	3.18	1	64	0	0	0	0	0	0	0	11013	9933
4/26/2021	2.07	1	54	0	0	0	0	0	0	0	11013	9933
4/27/2021	2.07	1	54	0	0	0	0	0	0	0	11013	9933
4/28/2021	2.07	1	54	0	0	0	0	0	0	0	11013	9933
4/29/2021	2.07	1	54	89	85	1	89	1	0	0	11102	10018
4/30/2021	2.72	1	57	41	37	1	41	1	0	0	11143	10055
5/3/2021	3.01	1	64	(13)	(17)	1	0	0	-13	-13	11130	10038
5/4/2021	1.94	1	20	0	0	0	0	0	0	0	11130	10038
5/5/2021	1.94	1	20	0	0	0	0	0	0	0	11130	10038
5/6/2021	1.94	1	20	0	0	0	0	0	0	0	11130	10038
5/7/2021	1.94	1	20	0	0	0	0	0	0	0	11130	10038
5/10/2021	1.94	1	20	(152)	(156)	1	0	0	-152	-152	10978	9882
5/11/2021	0.90	2	-1	124	112	3	168	1	-36	-44	11102	9994
5/12/2021	1.29	4	-15	0	0	0	0	0	0	0	11102	9994
5/13/2021	1.29	4	-15	324	320	1	324	1	0	0	11426	10314
5/14/2021	2.02	4	6	266	262	1	266	1	0	0	11692	10576
5/17/2021	3.05	4	29	64	60	1	64	1	0	0	11756	10636
5/18/2021	3.26	4	45	0	0	0	0	0	0	0	11756	10636
5/19/2021	3.26	4	45	70	58	3	138	2	-68	-68	11826	10694
5/20/2021	3.03	4	70	0	0	0	0	0	0	0	11826	10694
5/21/2021	3.94	4	74	(55)	(59)	1	0	0	-55	-55	11771	10635
5/24/2021	3.28	4	75	46	42	1	46	1	0	0	11817	10677
5/25/2021	3.42	4	76	0	0	0	0	0	0	0	11817	10677
5/26/2021	3.42	4	76	0	0	0	0	0	0	0	11817	10677
5/27/2021	3.42	4	76	(80)	(84)	1	0	0	-80	-80	11737	10593
5/28/2021	2.54	4	74	(63)	(67)	1	0	0	-63	-63	11674	10526
5/31/2021	2.12	4	69	0	0	0	0	0	0	0	11674	10526
6/1/2021	2.18	3	68	(218)	(230)	3	1	1	-123	-218	11456	10296
6/2/2021	1.48	3	36	0	0	0	0	0	0	0	11456	10296
6/3/2021	1.48	3	36	(163)	(167)	1	0	0	-163	-163	11293	10129
6/4/2021	1.19	3	23	0	0	0	0	0	0	0	11293	10129
6/7/2021	1.19	3	23	0	0	0	0	0	0	0	11293	10129
6/8/2021	1.46	3	14	0	0	0	0	0	0	0	11293	10129
6/9/2021	1.29	3	-23	0	0	0	0	0	0	0	11293	10129
6/10/2021	1.29	3	-23	217	209	2	217	2	0	0	11510	10338
6/11/2021	1.13	3	-67	(153)	(157)	1	0	0	-153	-153	11357	10181

Date	IS pf	IS lr	IS r2	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
6/14/2021	0.58	3	-80	0	0	0	0	0	0	0	11357	10181
6/15/2021	0.50	3	-84	0	0	0	0	0	0	0	11357	10181
6/16/2021	0.50	3	-84	25	17	2	47	1	-22	-22	11382	10198
6/17/2021	0.41	3	-76	0	0	0	0	0	0	0	11382	10198
6/18/2021	0.41	3	-76	(94)	(106)	3	0	0	-75	-94	11288	10092
6/21/2021	0.39	3	-72	458	454	1	458	1	0	0	11746	10546
6/22/2021	0.91	3	-18	0	0	0	0	0	0	0	11746	10546
6/23/2021	0.91	3	-18	0	0	0	0	0	0	0	11746	10546
6/24/2021	0.91	3	-18	139	135	1	139	1	0	0	11885	10681
6/25/2021	1.21	3	1	136	132	1	136	1	0	0	12021	10813
6/28/2021	1.53	3	20	0	0	0	0	0	0	0	12021	10813
6/29/2021	1.53	3	20	2	(6)	2	73	1	-71	-71	12023	10807
6/30/2021	2.13	3	67	0	0	0	0	0	0	0	12023	10807
7/1/2021	2.13	3	67	0	0	0	0	0	0	0	12023	10807
7/2/2021	3.15	3	67	0	0	0	0	0	0	0	12023	10807
7/5/2021	3.15	3	67	0	0	0	0	0	0	0	12023	10807
7/6/2021	3.15	3	67	0	0	0	0	0	0	0	12023	10807
7/7/2021	3.15	3	67	0	0	0	0	0	0	0	12023	10807
7/8/2021	3.15	3	67	255	247	2	255	2	0	0	12278	11054
7/9/2021	3.26	3	85	259	255	1	259	1	0	0	12537	11309

## Appendix I: $n^{\text{th}}$ Order Polynomial Next Bar's Forecast Math

### What is the $n^{\text{th}}$ Order Polynomial ?

The  $n^{\text{th}}$  Order Polynomial, also called the  $n^{\text{th}}$  Order Fixed Memory Polynomial, is simply the least square fit of a polynomial of the form  $b_0 + b_1 * t + b_2 * t^2 + b_3 * t^3 + \dots + b_n * t^n$  to a *fixed* number of past data points. Where  $t$  is discrete time bars. Time could be daily bars or 5-minute bars. We use the term "Fixed Memory" to designate that only a fixed number of data points are used to calculate the polynomial coefficients. It is assumed that the time bars occur at fixed intervals of time so tic bars would not be appropriate for this analysis. Least squares is a mathematical technique where the squared vertical distance between the data and the curve that is being fit to the data is minimized. When the net squared distance (also called the sum of the squared errors) is minimized, a unique set of coefficients  $b_0, b_1, b_2, \dots, b_n$  for the polynomial is determined. This type of error minimization is mathematically solvable and is widely used in science and mathematics.

For a  $4^{\text{th}}$  order polynomial equation, the least squares coefficients are obtained from the solution of the following matrix equation.

$$\begin{bmatrix} T & \sum t & \sum t^2 & \sum t^3 & \sum t^4 & a_0 & \sum p(t) \\ \sum t & \sum t^2 & \sum t^3 & \sum t^4 & \sum t^5 & b_0 & \sum (p(t) * t) \\ \sum t^2 & \sum t^3 & \sum t^4 & \sum t^5 & \sum t^6 & c_0 & \sum (p(t) * t^2) \\ \sum t^3 & \sum t^4 & \sum t^5 & \sum t^6 & \sum t^7 & d_0 & \sum (p(t) * t^3) \\ \sum t^4 & \sum t^5 & \sum t^6 & \sum t^7 & \sum t^8 & e_0 & \sum (p(t) * t^4) \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

where

$p(T)$  is the current bar's price,  $p(T-1)$  is the previous bar's price and  $p(1)$  is the price  $T$  bars ago.

$T$  is the number of Bars in the Least Squares estimation

$\sum p(t)$  is the summation of prices from  $t=1$  to  $T$  bars

$\sum p(t) * t$  is the summation of prices times  $t$  from  $t=1$  to  $T$  bars

$\sum t$  is the summation of the integer  $t$  from  $t=1$  to  $T$  bars

$\sum t^2$  is the summation of the integer  $t$  squared from  $t=1$  to  $T$  bars

etc.

Once the coefficients to the polynomial have been solved for, we generate the forecast for the next bar's price which is given for the equation by:

$$P_f = a_0 + b_0 * (T+1) + c_0 * (T+1)^2 + d_0 * (T+1)^3 + e_0 * (T+1)^4$$

Where  $P_f$  stands for price forecast.

With these coefficients, we can also generate the forecast for the next bar's *velocity* and *velocity* by the equations:

$$\text{Velocity}(T+1) = dP_f / dt = b_0 + 2c_0 * (T+1) + 3d_0 * (T+1)^2 + 4e_0 * (T+1)^3$$

$$\text{Velocity}(t+1) = d^2 P_f / d^2 t = 2c_0 + 6d_0 * (T+1) + 12e_0 * (T+1)^2$$

We use the next bar forecast because changes in the trend are more quickly reflected in the forecast price, velocity and velocity than in the end point price, velocity and velocity.

Programs that solve for the solution to matrix equations can be found in the book "Numerical Recipes" by W. Press, et. al. However this type of matrix equation solvers are very slow and for these types of problems are unstable. They cause numerical errors and floating point overflows due to matrix inversion ill conditioning which produces false results.

## Appendix I: $n^{\text{th}}$ Order Polynomial Next Bar's Forecast Math

Fortunately, these types of problems can be solved by a fast, efficient and accurate algorithm using Discrete Orthogonal Legendre Polynomials. This method is explained in detail in Norman Morrison's book entitled "Introduction to Sequential Smoothing and Prediction", Chapter 7 page 223., referenced at the end of this section.

Without going into detail here (see Morrison reference), the polynomial filter can now be represented by:

$$P_e(t) = \sum_{j=0}^n \beta_j \phi_j(t) \quad j=0 \text{ to } n$$

Where  $n$  is the polynomial order,  $T$  is the total number of Bars of data used in the Least Squares fit and

$$\beta_j = \sum_{k=0}^{T-1} p(t-T+k) \phi_j(k)$$

$\phi_j(t)$  = the *normalized discrete Legendre polynomial*.  $t$  = an integer from 0 to  $T$

The coefficients,  $\beta_0, \beta_1, \beta_2, \beta_3, \dots, \beta_n$  for a  $n^{\text{th}}$  order polynomial can now be solved for by the equation above and we can generate the forecast for the next bar's close, velocity and velocity which are given by the equations

$$P_F(T+1) = \beta_0 \phi_0(T+1) + \beta_1 \phi_1(T+1) + \beta_2 \phi_2(T+1) + \beta_3 \phi_3(T+1) + \dots + \beta_n \phi_n(T+1)$$

$$\text{Velocity} = (dP_F/dt)_{(T+1)} = \beta_1 (d\phi_1/dt)_{(T+1)} + \beta_2 (d\phi_2/dt)_{(T+1)} + \beta_3 (d\phi_3/dt)_{(T+1)} + \dots + \beta_n (d\phi_n/dt)_{(T+1)}$$

$$\text{Velocity} = (d^2P_F/d^2t)_{(T+1)} = \beta_2 (d^2\phi_2/d^2t)_{(T+1)} + \beta_3 (d^2\phi_3/d^2t)_{(T+1)} + \dots + \beta_n (d^2\phi_n/d^2t)_{(T+1)}$$

### The $n^{\text{th}}$ Order Fixed Memory Forecast Next Bar's Velocity Strategy Defined

The least squares forecast is constructed by solving for the least squares coefficients  $\beta_1, \beta_2, \dots, \beta_n$  at each bar using the last  $T$  bars of closing prices and the Discrete Orthogonal Legendre Polynomial equations for  $\beta_j$  above. Then **Velocity** =  $d^2P_F(T+1)/d^2t$  is constructed from the velocity equation above and plotted under the price chart. In general, what we will be doing is following the plotted curve of **Velocity** which is calculated at each bar from the previous  $T$  bars. When the velocity is greater than a threshold amount **vup** we will go long. When the velocity is less than a threshold amount **-vdown** we will go short.

#### **Buy Rule:**

IF **Velocity** is greater than the threshold amount **vup** then buy at the market.

#### **Sell Rule:**

IF **Velocity** is less than the threshold amount **-vdown** then sell at the market.

### References

1. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.

# The Normalization Multiplier

## What is the Multiplier?

The  $n^{\text{th}}$  Order Fixed Memory Polynomial, also called an  $n^{\text{th}}$  Order Polynomial, is the least square fit of a polynomial of the form  $b_0 + b_1 * t + b_2 * t^2 + b_3 * t^3 + \dots + b_n * t^n$  to a *fixed* number of past data points. Where  $t$  is discrete time bars. Time could be daily bars or 5-minute bars. We use the term "Fixed Memory" to designate that only a fixed number of data points are used to calculate the polynomial coefficients. It is assumed that the time bars occur at fixed intervals of time so tic bars would not be appropriate for this analysis. Least squares is a mathematical technique where the squared vertical distance between the data and the curve that is being fit to the data is minimized. When the net squared distance (also called the sum of the squared errors) is minimized, a unique set of coefficients  $b_0, b_1, b_2, \dots, b_n$  for the polynomial is determined. This type of error minimization is mathematically solvable and is widely used in science and mathematics. Once the  $b_n$  coefficients are found using a lookback period of  $T$  bars to calculate the  $b_n$  coefficients, then the next bar's estimate ( $T+1$ ) of the  $n^{\text{th}}$  order polynomial velocity and acceleration can be easily found by the equations below.

$$\text{Velocity}(T+1) = dP_f / dt = b_0 + 2c_0 * (T+1) + 3d_0 * (T+1)^2 + 4e_0 * (T+1)^3 + \dots + n * b_n * (T+1)^{n-1}$$

$$\text{Acceleration}(t+1) = d^2P_f / d^2t = 2c_0 + 9d_0 * (T+1) + 12e_0 * (T+1)^3 + \dots + n * (n-1) * b_n * (T+1)^{n-2}$$

Please see the  $n^{\text{th}}$  Order Fixed Memory Polynomial Next Bar's Forecast Math section for a more detailed explanation.

For any tradable, the inputs to the polynomial are the **polynomial degree (Order)** and the number or lookback bars  $N$  (denoted by  $T$  in equations above). When we plot the velocity or acceleration, we notice that the amplitude, and the maximum and minimum values of the velocity or acceleration vary quite significantly with different degree and  $N$  inputs.

Below is a table of the standard deviation (SD) of the 56340 calculated Velocity values for different **degree** and **N** inputs. We used 1min bars of the E-Mini from 8/1/2014 to 2/20/2015 to generate this table. As one can see the standard deviation of the velocity for each degree and  $N$  vary greatly. For instance, for degree=4,  $N=20$  the SD is 6.8 times the SD for degree=1,  $N=20$ . This creates problems when trying to determine the correct ranges for  $v_{up}/v_{dn}$  and  $a_{up}/a_{dn}$  during optimization.

**@ES.D 5 min bars Date Range 1140801 to 1150220**  
**Total Number of Bars=56340 Norm=0**  
**FixmVn Multiplier= 1/SD to Scale Velocity pw and N Range to One SD**

Degree	N	SD	1/SD
1	20	0.1902	5.2565
1	30	0.1540	6.4916
1	40	0.1328	7.5279
1	50	0.1183	8.4502
1	60	0.1077	9.3990
1	70	0.0996	10.0440
avg		0.1338	7.8430
2	20	0.4351	2.2982
2	30	0.3443	2.9046
2	40	0.2936	3.4060
2	50	0.1583	3.8275
2	60	0.2371	4.2180
2	70	0.2173	4.6010

## The Normalization Multiplier

avg		0.2981	3.5425
3	20	0.7854	1.2732
3	30	0.5933	1.6855
3	40	0.4973	2.0111
3	50	0.4347	2.3005
3	60	0.3949	2.5324
3	70	0.3656	2.7352
avg		0.5119	2.0897
4	20	1.2924	0.7738
4	30	0.9279	1.0777
4	40	0.7582	1.3189
4	5	0.6542	1.5285
4	60	0.5804	1.7228
4	70	0.5314	1.8818
avg		0.7908	1.3839

The problem may get worse when we want to find good inputs for other tradables. Other tradables, because of their scales and tick size have much different Velocity ranges than the E-Mini for the same degree and N. Thus, the NS search ranges have to be different for each different tradable.

To solve this problem and to have a standard search space for each tradable, I created a **Mult** input for each FixmXVA Velocity and Acceleration strategy and indicator. If each tradable's Velocity is multiplied by a number such that the standard deviation of that tradable's Velocity is around one, then the search space for vup and vdn for each tradable would be 0 to 3.5 SDs and we wouldn't have to change the TS search space every time we wanted to examine a new stock or future. The complicated equations that I use to normalize the ranges to one standard deviation were derived using the software TableCurve 3D, automated surface and equation discovery.