The Fading Memory Polynomial Forecast Price Strategy Applied To 1Min bar e-Mini Futures from June/2012 –May/2015

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This is a mathematical technique that fits a n^{th} order polynomial to the last N price bars but calculates the coefficients of the polynomial such that the error between the current n^{th} order polynomial and the current price bar is weighted much higher than the error between the price N bars ago Consider a time series x(t) where t is an integer value (a price bar number) like the number of days or minutes, etc from some starting time. Suppose we want to find at some given time some *n*th-degree polynomial that fits the data well at current and recent prices but ignores the fit as we move into the distant past. This is what the Fading Memory Polynomial does (see Appendix 1). As an example, if the latest price is at time t and the price made a turn at time bar t-10, then we do not want prices prior to t-10 affecting the current polynomial fit as much. As will be shown the most familiar case of the fading memory technique is the exponential moving average, which is a Fading Memory Polynomial of Order 1. The fading memory technique is in contrast to the Least Squares Polynomial fit, which weights all past errors between the polynomial and the price bar equally.

In a previous working paper entitled "The Fading Memory Polynomial Velocity Strategy Applied To 1Min bar Euro Futures from Jan/2008 –Dec/2013" we showed how the application of a velocity curve generated by Nth Order Fixed Memory Polynomial Velocity could be used to develop a strategy to buy and sell the Euro future intraday. The reasoning behind this type of system was to only trade when the price trend velocity was above a certain threshold.

Here we will use a technique called the Fading Memory Polynomial Percent Movement Strategy to trade the E-Mini futures contract. Let us define the Fading Memory Polynomial inputs pctup and pctdn as follows: The Fading Memory Polynomial generates an estimate of the next bars price. Let us call that estimate $P_f(t+1)$, for polynomial forecast. . If $\mathbf{P_f(t+1)}$ has moved up by more than the percentage amount of *pctup* from the lowest low recorded in $\mathbf{P_f(t+1)}$ while short then we would buy one E-Mini contract at the market. If $\mathbf{P_f(t+1)}$ has moved down by more than the percentage amount *pctdn* from the highest high recorded in $\mathbf{P_f(t+1)}$ while long then sell one E-Mini contract at the market. What we are doing is following the $\mathbf{P_f(t+1)}$ curve and buying when the curve moves up a certain percentage and selling when the curve moves down a certain percentage. The reasoning behind this type of system was to only trade when the price trend moved above a certain threshold. Many times prices meander around without any notable trend and this is considered noise. During these times we do not wish to trade because of the cost of whipsaw losses that would occur from this type of price action. When a price trend finally starts, the movement of that price trend moves above a minimum threshold noise value. Thus the Fading Memory Polynomial percent movement strategy would only issue a trade when certain percentage thresholds above "noise" levels are crossed.

The Nth Order Fading memory Polynomial Percent Movement Strategy has a number of unknown inputs that we have to determine before we can use this strategy to trade. The unknown inputs to the Fading Memory Polynomial are the polynomial order(degree), the weighting factor to weight the past errors with a number that got smaller and smaller the further back in time we went and finally the percent up(pctup) and percent down(pctdn) movements thresholds.

Here we will use Walk Forward Optimization and out-of-sample performance to determine the "best" polynomial inputs as well as how these inputs should change over time. We will use the nth Order Fading Memory Adaptive Polynomial Percent Movement System to trade the E-Mini futures contract on an intraday basis using one minute bar price data. To test this strategy we will use one minute bar prices of the E-Mini futures contract(ES) traded on the CME/Globex from May 1, 2012 to June1, 2015.

The n^{th} Order Fading Memory Adaptive Polynomial Defined

The adaptive n^{th} order Fading Memory Polynomial is constructed and plotted at each bar by solving for the coefficients $\mathbf{b_1, b_2, b_3, \ldots b_n}$ for the discrete orthogonal Meixner polynomials at each bar using the exponential decay factor $\boldsymbol{\alpha=(1-\beta)}$ and the equation for $\mathbf{b_j}$ shown in the **"Math"** appendix of this paper. Then $\mathbf{P_f(T+1)}$ is constructed from the equation shown in the "Math" appendix and plotted with the price chart.

The $P_f(t+1)$ of a 2nd and 3rd order polynomial should change faster than the straight line (1st order). As observed from the 2nd order price equation in the "Math" section, there is an acceleration component in the calculation of the polynomial price. This means that the 2nd order will reflect a change in the price trend much faster than the straight line estimation which does not have an acceleration component. The same is true for 3rd and 4th order polynomial price estimations. Whether higher order polynomial velocities is an advantage or not we will let the computer decide when we let the computer search for the "best" polynomial degree as described below.

At each bar we calculate the nth order (1st through 3rd) fading memory polynomial from the formulas in the "Math" appendix. As we will show below, optimization will determine the order for nth order polynomial that will be used. When $P_f(t+1)$ has moved up by more than the percentage amount of *pctup* from the lowest low recorded in $P_f(t+1)$ while short then buy one E-Mini contract at the market.
When $P_f(t+1)$ has moved down by more than the percentage amount *pctdn* from the highest high recorded in $P_f(t+1)$ while long then sell one E-Mini contract at the market.

Buy Rule:
If $P_f(t+1)$ has moved up by more than the percentage amount of *pctup* from the lowest low recorded in **$P_f(t+1)$** while short then buy one E-Mini contract at the market.
Sell Rule:
If $P_f(t+1)$ has moved down by more than the percentage amount *pctdn* from the highest high recorded in **$P_f(t+1)$** while long then sell one E-Mini contract at the market.

Intraday Bars Exit Rule:
Close the position at 1500(CST) before the ES close (no trades will be carried overnight).

Intraday Bars First Trade of Day Entry Rule:

Ignore all trade signals before **9:00**am(CST). For the Buy and Sell rules above we have included a first trade of the day entry rule. Trading in the ES futures has changed a lot in the last 4 years because of 24hr Globex trading. In particular trading starts a lot earlier in the morning when Asia and then Europe opens and then dies down. The NYSE opens at 8:30am and closes at 3pm. We will only trade the day ES while the NYSE is open.

Discussion of E-Mini Prices

The E-Mini(ES) is traded on Globex and on the trading floor at the CME. On Globex the ES is traded on a 23hour basis . The CME hours for floor trading (RTH) are 8:30 to 15:15 CST. We have restricted our study to only trading the ES during the 8:30 to 15:00 hours.

Testing The Polynomial Velocity System Using Walk Forward Optimization

There will be four strategy parameters to determine:

1. **degree**, degree=1 for straight line velocity, degree=2 for 2nd order velocity, etc.
2. **N = where $-\beta = 1-2/(N+1)$ β** is the exponential decay weight for the Nth Order Fading Memory Polynomial calculation**.**
3. ***pctup***, the percent amount *up* from the lowest low recorded in **Pf(t+1)** while short
4. ***pctdn***, the percent amount *down* from the highest high recorded in **Pf(t+1)** while long

To test this system we will use one minute bar prices of the E-Mini(ES) futures contract traded on the CME/Globex and known by the symbol ES for the 156 weeks from May 1, 2012 to May 1, 2015.

We will test this strategy with the above ES 1 min bars on a walk forward basis, as will be described below. To create our walk forward files we will use the ***add-in*** software product called the Power Walk Forward Optimizer (PWFO). In TradeStation (TS), we will run the PWFO strategy ***add-in*** along with the nth Order Fading Memory Polynomial Strategy on the ES 1min data from May 1, 2012 to May 1, 2015. The PWFO will breakup and create 30 day calendar in-sample sections along with their corresponding one calendar week out-of-sample sections from the 156 weeks of ES (see Walk forward Testing below) creating 152 out-of-sample weeks.

What Is An In-Sample Section and Out-Of-Sample Section?

Whenever we do a TS optimization on a number of different strategy inputs, TS generates a report of performance metrics (total net profits, number of losing trades, etc) vs these different inputs. If the report is sorted on say the total net profits(***tnp***) performance metric column then the highest ***tnp*** would correspond to a certain set of inputs. This is called an ***in-sample*** or ***test section***. If we choose a set of strategy inputs from this report based upon some performance metric we have no idea whether these strategy inputs will produce the same results on future price data or data they have not been tested on. Price data that is not in the in-sample section is defined as ***out-of-sample data***. Since the performance metrics generated in the in-sample section are mostly due to "curve fitting" (see Walk Forward Out-of-Sample Testing section below) it is important to see how the strategy inputs chosen from the in-sample section perform on out-of-sample data.

What Does The Power Walk Forward Optimizer (PWFO) Do?

The PWFO is a TS ***add-in*** that breaks up the TS optimization run into a number of user selectable in-sample and out-of-sample sections. The PWFO prints out the in-sample sample performance metrics **and the out-of-sample performance results**, on one line, for each case or input variable combination that is run by the TradeStation(TS) optimization module to a user selected spreadsheet comma delimited file. The PWFO can generate up to 500 different in-sample and out-of-sample date optimization files in one TS run, saving the user from having to generate optimization runs one at a time. The PWFO output allows you to quickly determine whether your procedure for selecting input parameters for your strategy just curve fits the price and noise, or produces statistically valid out-of-sample results. In addition to the out-of-sample performance results presented for each case, 30+ superior and robust performance metrics (many are new and never presented before) are added to each case line in the in-sample section and printed out to the comma delimited file. These 30+ performance metrics allow for a superior and robust selection of input variables from the in-sample section that have a higher probability of performing well on out-of-sample data (Please see Appendix 2 for a listing of these performance metrics).

For our computer run we will have the PWFO breakup the 156 weeks of ES one minute bar price data into 152 in-sample/out-of sample files. The in-sample sections will be 30 calendar days and the out-of-sample(oos) section will be the one week following the in-sample section. The oos week will always end on a Friday as will the 30 day calendar in-sample section. As an example the first in-sample section would be from 5/3/2012 to 6/1/2012 and the out-of-sample section would be the week following from 6/4/2012 to 6/8/2012.(our in-sample and out-of-sample sections always end on a Friday). We would then move everything ahead a week and the 2nd in-sample section would be from 5/10/2012 to 6/8/2012 and the week following out-of-sample section would be from 6/11/2012 to 6/15/2012. Etc.

The PWFO 152 in-sample/out-of-sample section dates are shown in **Table 1** on page 8 below. We will then use another software product called the Walk Forward Performance Metric Explorer (WFME) on each of the 152 in-sample and out-of-sample(oos) sections generated by the PWFO to find the best in-sample section performance ***filter*** that determines the system input parameters ***(degree, N, pctup, pctdn)*** that will be used on the out-of-sample data. Detailed information about the PWFO and the WFME can be found at www.meyersanalytics.com

For the in-sample data we will run the TradeStation optimization engine on the 314 weeks of ES 1 min bars with the following ranges for the nth order fading memory polynomial velocity strategy input variables.

1. degree from 1 to 3
2. N from 20 to 80 in steps of 20.
3. pctup from 0.2 to 1 steps of 0.2
4. pctdn from 0.2 to 1in steps of 0.2

Note: I use **N** because it gives a better understanding of how many bars of past data are approximately being used. **N** and α **(α=1-β)** are approximately related by the formula **α=2/(1+N)**. N is converted to α by this formula in the Nth Order Fading Memory Polynomial calculation

This will produce 300 different cases or combinations of the input parameters for each of the 152 PWFO output files.

Walk Forward Out-of-Sample Testing

Walk forward analysis attempts to minimize the curve fitting of price noise by using the law of averages from the Central Limit Theorem on the out-of-sample performance. In walk forward analysis the data is broken up into many in-sample and out-of-sample sections. Usually for any system, one has some performance metric selection procedure, which we will call a ***filter,*** used to select the input parameters from the optimization run. For instance, a ***filter*** might be all cases that have a profit factor (PF) greater than 1 and less than 3. For the number of cases left, we might select the cases that had the best percent profit. This procedure would leave you with one case in the in-sample section output and its associated strategy input parameters. Now suppose we ran our optimization on each of our many in-sample sections and applied our filter to each in-sample section output. We would then use the strategy input parameters found by the ***filter*** in each in-sample section on the out-of-sample section immediately following that in-sample section. The input parameters found in each in-sample section and applied to each out-of-sample section would produce independent net profits and losses for each of the out-of-sample sections. Using this method we now have "x" number of independent out-of-sample section profit and losses from our filter. If we take the average of these out-of-sample section net profits and losses, then we will have an estimate of how our system will perform on average. Due to the Central Limit Theorem, as our sample size increases, the spurious noise results in the out-of-sample section performance tend to average out to zero in the limit leaving us with what to expect from our system and filter. ***Mathematical note:*** This assumption assumes that the out-of-sample returns are from probability distributions that have a finite variance.

Why use the walk forward technique? Why not just perform an optimization on the whole price series and choose the input parameters that give the best total net profits or profit factor? Surely the price noise cancels itself out with such a large number of in-sample prices and trades. Unfortunately, nothing could be farther from the truth! Optimization is a misnomer and should really be called combinatorial search. As stated above, whenever we run a combinatorial search over many different combinations of input parameters on noisy data on a fixed number of prices, ***no matter how many***, the best performance parameters found are guaranteed to be due to ***"curve fitting"*** the noise and signal. What do we mean by ***"curve fitting"***? The price series that we trade consists of random spurious price movements, which we call noise, and repeatable price patterns (*if they exist*). When we run, for example, 5000 different input parameter combinations, the best performance parameters will be from those system input variables that are able to produce profits from the price pattern ***and*** the random spurious movements. While the price patterns will repeat, the same spurious price movements will not. If the spurious movements that were captured by a certain set of input parameters were a large part of the total net profits, then choosing these input parameters will produce losses when traded on future data. These losses occur because the spurious movements will not be repeated in the same way. This is why system optimization, neural net optimizations or combinatorial searches with no out-of-sample testing cause loses when traded in real time from something that looked great in the in-sample section. Unfortunately it is human nature to extrapolate past performance to project future trading results and thus results from curve fitting give the illusion, a modern "siren call" so to speak, of future trading profits.

In order to gain confidence that our input parameter selection method using the optimization output of the in-sample data will produce profits, we must test the input parameters we found in the in-sample section on out-of-sample data. In addition, we must perform the in-sample/out-of-sample analysis many times. Why not just do the out-of-sample analysis once? Well just as in Poker or any card game, where there is considerable variation in luck from hand to hand, walk forward out-of-sample analysis give considerable variation in week to week out-of-sample profit "luck". That is, by pure chance we may have chosen some input parameter set that did well in the in-sample section data ***and*** the out-of-sample section data. In order to minimize this type of "luck", statistically, we must repeat the walk forward out-of-sample (oos) analysis over many in-sample/oos sections and take an average of our weekly results over all out-of-sample sections. This average gives us an expected weekly return and a standard deviation of weekly returns which allows us to statistically estimate the expected equity and its range for N weeks in the future.

Finding The Strategy Input Parameters in The Walk Forward In-Sample Sections

The PWFO generates a number of performance metrics in the in-sample section. (Please see appendix II for a listing of these performance metrics). The question we are attempting to answer statistically, is which performance metric or combination of performance metrics (which we will call a ***filter***) in the in-sample section will produce strategy inputs that produce statistically valid profits in the out-of-sample section. In other words we wish to find a metric ***filter*** that we can apply to the in-sample section that can give us strategy inputs that will produce, on average, good trading results in the future. The PWFO produces a total of 32 different performance metrics in the in-sample section. If we have 300 different input variations or cases then the in-sample section consists of 32 columns of performance metrics for each of the 300 input cases or rows.

An example of a simple filter would be to choose the row in the in-sample section that had the highest net profit or perhaps a row that had one the best performance metric from one of the other 32 PWFO metrics. Unfortunately it was found that this type of simple filter very rarely produces good out-of-sample results. More complicated metric filters can produce good out-of-sample results minimizing spurious price movement biases in the selection of strategy inputs.

Here is an *example* of a more complicated ***filter*** that was used in this paper. We fit the equity curve by a 2nd order polynomial instead of a straight line and the metric, **eq2A,** is the acceleration of that 2nd order polynomial. Let us choose the 20 rows in the in-sample section that contain the **largest(top) 20 eq2A** values. In other words we sort the metric **eq2A** from high to low, in the in-sample section and then choose the top 20 Rows .. Let us choose the 20 rows in the in-sample section that contain the **largest(top) 20 eq2A** values. In other words we sort the metric **eq2A** from high to low, in the in-sample section and then choose the top 20 Rows .. This particular filter will now leave 20 cases or rows in the in-sample section. We call this part of the filter **t20eq2A**.. Suppose for this filter, within the 20 in-sample rows that are left, we want the row that has the minimum metric called **mWT|LT** . This metric is the Ratio of Median Winning Trades to Median Losing Trades. We use the median rather than the average so that a few outliers won't distort this metric. Thus, we would want the median to be as large as possible. This would produce a final filter named **t20eq2A-mWT|LT**. We abbreviate this final filter as **t20eq2A-mWT|LT**. For each in-sample section this filter leaves only one row in the in-sample section with its associated strategy inputs and out-of-sample net profit in the out-of-sample section using the strategy inputs found in the in-sample section. This particular **t20eq2A-mWT|LT filter** is then applied to each of the 152 in-sample sections which give 152 sets of strategy inputs that are used to produce the corresponding 152 weeks of out-of-sample performance results. The average out-of-sample performance is calculated from these 152 weeks of out-of-sample performance results. In addition many other important out-of-sample performance statistics for this filter are calculated and summarized. **Figure 3** shows such a computer run along with a small sample of other filter combinations that are constructed in a similar manner. Row 3 of the sample output in **Figure 3** shows the results of the filter discussed above.

Bootstrap Probability of Filter Results: Using modern "Bootstrap" techniques, we can calculate the probability of obtaining each filter's total out-of-sample ***net*** profits by chance. By ***net*** we mean subtracting the cost and slippage of all round trip trades from the total out-of-sample profits. Here is how the bootstrap technique is applied. Suppose as an example, we calculate the total out-of-sample net profits(tOnpNet) over all out-of-sample weeks for a given filter like above. A mirror filter is created. However, instead of picking an out-of-sample net profit(OSNP) from a row that the filter picks, the mirror filter picks a ***random*** row's OSNP in each of the 152 PWFO files. Suppose we repeat this random row section 5000 times. Each of the 5000 mirror filters will choose a random row's OSNP of their own in each of the 152 PWFO files. At the end, each of the 5000 mirror filters will have 152 ***random*** OSNP's picked from the rows of the 152 PWFO files. The sum of the 152 random OSNP picks for each mirror filter will generate a random total out-of-sample net profit(tOnpNet). The average and standard deviation of the 5000 mirror filter's different random tOnpNets will allow us to calculate the chance probability for each ***our*** filter's tOnpNet. Thus given the mirror filter's bootstrap random tOnpNet average and standard deviation, we can calculate the probability of obtaining our filter's tOnpNet by pure chance alone. Since for this run we examined 1922(shown in Figure 3) different filters, we can calculate the expected number of cases that we could obtain by pure chance that would match or exceed the tOnpNet of the filter we have chosen or (1922) X (tOnpNet Probability). For our filter in row 3 in Figure 3 the expected number of cases that we could obtain by pure chance that would match or exceed the \$18,394 is $1922 \times 3.3\ 10^{-4} = 0.63$. This is less than one case so it is improbable that our result was due to pure chance.

Results

Table 1 on page 8 below presents a table of the 152 in-sample and out-of-sample windows, the selected optimum parameters and the weekly out-of-sample results using the filter described above.

Figure 1 presents a graph of the equity and net equity curves generated by using the filter on the 152 weeks ending 6/8/2012 to 5/1/2015. The equity curves are plotted from the Equity and Net Equity columns in Table 1. Plotted on the equity curves are 2nd Order Polynomial fits. The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity. The brown line is the net equity curve with commissions and the green dots are the new highs in net equity.

Figure 2 Walk Forward Out-Of-Sample Performance for ES Fading Memory Polynomial System
1 minute bar chart of ES from 4/30/15-5/1/15

Figure 3 Partial output of the Walk Forward Metric Performance Explorer (WFME)
Run on the 152 PWFO files of the ES 1min bars Nth Order Fading Memory System

Discussion of System Performance

In Figure 3 Row 4 of the spreadsheet filter output are some statistics that are of interest for our filter. **BE** is the break even weeks. Assuming the trade average and standard deviation for this filter are from a normal distribution, this is how many weeks we need to trade this strategy so that we have a 98% probability that the equity after that number of weeks will be greater than zero. BE is 34 weeks for this filter. This means we would have to trade this strategy for at least 34 weeks to have a 98% probability that our equity would be positive. Another interesting statistic is **Blw**. Blw is the maximum number of weeks the OSNP equity curve failed to make a new high. Blw is 10 weeks for this filter. This means that 10 weeks was the longest time that the equity for this strategy failed to make a new equity high.

To see the effect of walk forward analysis, take a look at **Table 1**. Notice how the input parameters ***degree, N, pctup and pctdn*** take sudden jumps from high to low and back. This is the walk forward process quickly adapting to changing volatility conditions in the in-sample sample. In addition, notice how often ***degree*** changes from a straight line velocity with ***degree=1*** to a 2nd and 3rd order polynomial. The 2nd and 3rd order polynomials, due to the higher order components, change much faster than the straight. When the data gets very noisy with a lot of spurious price movements, it's better to have the lower orders change slower filtering out the noisy data. During other times when the noise level is not as much it is better to have the higher order polynomial change ***pctup*** and ***pctdn*** faster to get onboard a trend faster. This is what the filter is doing. When there is a lot of noise in the in-sample section it switches to the 1st or 2nd order polynomial. When the noise level is lower in the in-sample section, it switches to the faster changing 3rd order polynomial velocity.

Using this filter, the strategy was able to generate $18,394net equity after commissions and slippage trading one ES contract for 152 weeks. Note $25 roundtrip commission and slippage was subtracted from each trade and no positions were carried overnight. The largest losing week was -$1838 and the largest drawdown was -$2976. The longest time between new equity highs was 10 weeks.

In observing Table 1 we can see that this strategy and filter made trades from a low of one trade/week to a high of 23 trades/week with an average of 7.4 trades/week. The strategy seemed to wait for really strong trends and then initiate a buy or sell. Out of the 152 out-of-sample weeks the filter traded all weeks with 59% of all trades profitable. In observing the chart from 11/7/2013 we can see the strategy trading mostly only when there is a big trend action.

Given 24 hour trading of the E-Mini, restricting the strategy to trade only from 830am to 3pm caused the strategy to miss many profitable trends opportunities when Asia and then Europe opened trading in the early morning. Further research will include the A.M. time zones.

<u>**References**</u>

1. Efron, B., Tibshirani, R.J., (1993), "An Introduction to the Bootstrap", New York, Chapman & Hall/CRC.
2. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.

Table 1 Walk Forward Out-Of-Sample Performance Summary for ES Nth Order Fading Memory Polynomial Velocity System

ES-1 min bars 5/2/2012 - 5/1/2015. The input values *degree(pw), N, pctup, pctdn* are the values found from applying the filter to the in-sample section optimization runs.

Filter= top 20 eq2A and then minimum mWT|LT

osnp = Weekly Out-of-sample gross profit in $
Equity = Running Sum of weekly out-of-sample gross profits $
NOnp$25 = Weekly Out-Of-Sample Net Profit in $ = **osnp-ont*25**.
NetEq = running sum of the weekly out-of-sample net profits in $
ollt = The largest losing trade in the out-of-sample section in $.
odd = The drawdown in the out-of-sample section in $.
ont = The number of trades in the out-of-sample week.
degree, degree=1 for straight line velocity, degree=2 for 2^{nd} order velocity, etc.
len = N the lookback period
pctup, the threshold amount that velocity has to be greater than to issue a buy signal
pctdn, the threshold amount that velocity has to be less than to issue a sell signal
Note: Blank rows indicate that no out-of-sample trades were made that week

In-Sample Dates				Out_Of_Sample Dates			osnp	NOnp$25	NetEq	ollt	odd	ont	degree	NDays	pctup	pctdn
05/02/12	to	06/01/12		06/04/12	to	06/08/12	-1213	-1488	-1488	-375	-1625	11	1	60	0.6	0.2
05/09/12	to	06/08/12		06/11/12	to	06/15/12	-113	-313	-1801	-438	-788	8	2	60	0.8	1
05/16/12	to	06/15/12		06/18/12	to	06/22/12	-113	-338	-2139	-513	-1513	9	1	40	0.4	1
05/23/12	to	06/22/12		06/25/12	to	06/29/12	138	-37	-2176	-338	-625	7	3	80	1	0.6
05/30/12	to	06/29/12		07/02/12	to	07/06/12	-150	-350	-2526	-363	-600	8	3	60	0.4	0.6
06/06/12	to	07/06/12		07/09/12	to	07/13/12	150	-25	-2551	-650	-650	7	1	80	0.2	1
06/13/12	to	07/13/12		07/16/12	to	07/20/12	225	25	-2526	-238	-450	8	2	20	0.8	0.4
06/20/12	to	07/20/12		07/23/12	to	07/27/12	1125	875	-1651	-338	-575	10	3	40	0.2	1
06/27/12	to	07/27/12		07/30/12	to	08/03/12	-25	-225	-1876	-263	-388	8	3	40	0.8	0.6
07/04/12	to	08/03/12		08/06/12	to	08/10/12	-463	-588	-2464	-413	-713	5	2	80	0.8	0.2
07/11/12	to	08/10/12		08/13/12	to	08/17/12	-938	-1113	-3577	-550	-938	7	2	40	1	0.2
07/18/12	to	08/17/12		08/20/12	to	08/24/12	-913	-1088	-4665	-563	-1150	7	1	20	0.8	1
07/25/12	to	08/24/12		08/27/12	to	08/31/12	200	50	-4615	-50	-100	6	1	60	0.2	0.8
08/01/12	to	08/31/12		09/03/12	to	09/07/12	1200	1075	-3540	-13	-13	5	1	60	0.2	0.8
08/08/12	to	09/07/12		09/10/12	to	09/14/12	700	525	-3015	-175	-175	7	3	40	1	0.4
08/15/12	to	09/14/12		09/17/12	to	09/21/12	-750	-950	-3965	-313	-750	8	1	40	0.2	1
08/22/12	to	09/21/12		09/24/12	to	09/28/12	163	13	-3952	-450	-538	6	1	60	1	0.8
08/29/12	to	09/28/12		10/01/12	to	10/05/12	-513	-663	-4615	-388	-738	6	1	80	0.4	1
09/05/12	to	10/05/12		10/08/12	to	10/12/12	1575	1450	-3165	-50	-50	5	2	20	0.8	0.2
09/12/12	to	10/12/12		10/15/12	to	10/19/12	225	25	-3140	-350	-775	8	2	20	0.8	0.2
09/19/12	to	10/19/12		10/22/12	to	10/26/12	550	425	-2715	-63	-63	5	1	80	1	0.8
09/26/12	to	10/26/12		10/29/12	to	11/02/12	-163	-238	-2953	-675	-675	3	1	20	1	1
10/03/12	to	11/02/12		11/05/12	to	11/09/12	588	413	-2540	-600	-1013	7	2	80	1	0.8
10/10/12	to	11/09/12		11/12/12	to	11/16/12	-400	-625	-3165	-425	-938	9	3	80	1	0.6
10/17/12	to	11/16/12		11/19/12	to	11/23/12	538	388	-2777	-213	-350	6	2	40	0.6	0.4
10/24/12	to	11/23/12		11/26/12	to	11/30/12	-413	-563	-3340	-425	-713	6	1	80	0.2	1
10/31/12	to	11/30/12		12/03/12	to	12/07/12	-263	-413	-3753	-563	-713	6	1	80	0.2	0.8
11/07/12	to	12/07/12		12/10/12	to	12/14/12	-438	-588	-4341	-213	-438	6	2	80	0.2	1
11/14/12	to	12/14/12		12/17/12	to	12/21/12	163	-12	-4353	-513	-1050	7	1	40	0.6	0.6
11/21/12	to	12/21/12		12/24/12	to	12/28/12	1513	1388	-2965	0	0	5	2	40	0.8	0.4
11/28/12	to	12/28/12		12/31/12	to	01/04/13	1425	1325	-1640	-25	-25	4	3	40	0.8	1
12/05/12	to	01/04/13		01/07/13	to	01/11/13	175	25	-1615	-138	-363	6	1	40	0.8	0.8
12/12/12	to	01/11/13		01/14/13	to	01/18/13	925	800	-815	0	0	5	3	60	0.2	0.8

In-Sample Dates				Out_Of_Sample Dates			osnp	NOnp$25	NetEq	ollt	odd	ont	degree	NDays	pctup	pctdn
12/19/12	to	01/18/13		01/21/13	to	01/25/13	375	275	-540	-163	-163	4	1	80	0.2	0.8
12/26/12	to	01/25/13		01/28/13	to	02/01/13	263	138	-402	-275	-488	5	3	60	0.2	1
01/02/13	to	02/01/13		02/04/13	to	02/08/13	225	75	-327	-238	-425	6	1	60	0.6	0.4
01/09/13	to	02/08/13		02/11/13	to	02/15/13	-175	-200	-527	0	0	1	3	20	1	0.2
01/16/13	to	02/15/13		02/18/13	to	02/22/13	450	300	-227	-238	-488	6	3	60	1	0.2
01/23/13	to	02/22/13		02/25/13	to	03/01/13	1750	1550	1323	-588	-588	8	1	80	0.8	0.2
01/30/13	to	03/01/13		03/04/13	to	03/08/13	-388	-538	785	-388	-388	6	2	60	0.8	0.2
02/06/13	to	03/08/13		03/11/13	to	03/15/13	600	475	1260	-125	-125	5	1	20	0.4	1
02/13/13	to	03/15/13		03/18/13	to	03/22/13	-800	-1025	235	-700	-1188	9	3	20	0.2	1
02/20/13	to	03/22/13		03/25/13	to	03/29/13	-63	-263	-28	-575	-1038	8	3	20	0.2	0.6
02/27/13	to	03/29/13		04/01/13	to	04/05/13	-963	-1113	-1141	-625	-963	6	3	60	1	0.6
03/06/13	to	04/05/13		04/08/13	to	04/12/13	-250	-475	-1616	-675	-1025	9	2	60	0.8	0.2
03/13/13	to	04/12/13		04/15/13	to	04/19/13	1700	1525	-91	-638	-1025	7	1	20	0.6	0.8
03/20/13	to	04/19/13		04/22/13	to	04/26/13	125	-25	-116	-313	-500	6	3	40	1	0.8
03/27/13	to	04/26/13		04/29/13	to	05/03/13	238	63	-53	-413	-463	7	2	80	0.6	0.6
04/03/13	to	05/03/13		05/06/13	to	05/10/13	413	288	235	-263	-263	5	2	60	1	1
04/10/13	to	05/10/13		05/13/13	to	05/17/13	1275	1150	1385	-413	-413	5	2	20	0.2	1
04/17/13	to	05/17/13		05/20/13	to	05/24/13	763	563	1948	-300	-338	8	2	60	0.2	1
04/24/13	to	05/24/13		05/27/13	to	05/31/13	38	-112	1836	-538	-813	6	2	80	1	0.8
05/01/13	to	05/31/13		06/03/13	to	06/07/13	1125	900	2736	-500	-600	9	3	40	1	0.2
05/08/13	to	06/07/13		06/10/13	to	06/14/13	813	563	3299	-713	-1088	10	3	40	0.8	0.2
05/15/13	to	06/14/13		06/17/13	to	06/21/13	2063	1763	5062	-338	-738	12	3	60	0.8	0.2
05/22/13	to	06/21/13		06/24/13	to	06/28/13	-838	-1013	4049	-625	-963	7	1	40	1	1
05/29/13	to	06/28/13		07/01/13	to	07/05/13	-1050	-1175	2874	-588	-1488	5	1	60	0.8	1
06/05/13	to	07/05/13		07/08/13	to	07/12/13	-1088	-1263	1611	-413	-1088	7	3	20	0.8	0.2
06/12/13	to	07/12/13		07/15/13	to	07/19/13	463	338	1949	-200	-225	5	3	20	1	1
06/19/13	to	07/19/13		07/22/13	to	07/26/13	350	225	2174	-300	-500	5	3	20	0.2	1
06/26/13	to	07/26/13		07/29/13	to	08/02/13	0	-125	2049	-250	-625	5	1	60	0.6	0.6
07/03/13	to	08/02/13		08/05/13	to	08/09/13	388	288	2337	-175	-175	4	1	80	1	0.2
07/10/13	to	08/09/13		08/12/13	to	08/16/13	538	388	2725	-200	-313	6	1	60	0.6	0.2
07/17/13	to	08/16/13		08/19/13	to	08/23/13	1000	850	3575	-250	-250	6	2	40	1	0.4
07/24/13	to	08/23/13		08/26/13	to	08/30/13	-500	-650	2925	-413	-838	6	3	60	1	0.8
07/31/13	to	08/30/13		09/02/13	to	09/06/13	988	838	3763	-288	-313	6	3	20	0.6	0.2
08/07/13	to	09/06/13		09/09/13	to	09/13/13	513	388	4151	-200	-200	5	3	20	0.6	0.6
08/14/13	to	09/13/13		09/16/13	to	09/20/13	538	388	4539	-350	-675	6	2	80	0.6	1
08/21/13	to	09/20/13		09/23/13	to	09/27/13	-225	-400	4139	-150	-525	7	3	20	0.2	0.8
08/28/13	to	09/27/13		09/30/13	to	10/04/13	-300	-450	3689	-425	-625	6	2	80	1	1
09/04/13	to	10/04/13		10/07/13	to	10/11/13	1913	1763	5452	-163	-300	6	2	20	0.8	0.6
09/11/13	to	10/11/13		10/14/13	to	10/18/13	1025	875	6327	-550	-550	6	2	20	0.8	0.6
09/18/13	to	10/18/13		10/21/13	to	10/25/13	-25	-175	6152	-213	-325	6	1	20	0.4	1
09/25/13	to	10/25/13		10/28/13	to	11/01/13	-813	-963	5189	-463	-1288	6	2	80	1	1
10/02/13	to	11/01/13		11/04/13	to	11/08/13	1100	825	6014	-288	-488	11	1	20	0.2	0.4
10/09/13	to	11/08/13		11/11/13	to	11/15/13	-600	-775	5239	-488	-713	7	3	60	0.8	0.2
10/16/13	to	11/15/13		11/18/13	to	11/22/13	938	788	6027	-163	-163	6	2	40	0.8	0.2
10/23/13	to	11/22/13		11/25/13	to	11/29/13	-313	-463	5564	-213	-475	6	1	20	0.6	0.2
10/30/13	to	11/29/13		12/02/13	to	12/06/13	-1063	-1288	4276	-500	-1575	9	2	20	0.6	0.8
11/06/13	to	12/06/13		12/09/13	to	12/13/13	600	475	4751	-163	-300	5	3	80	1	0.6
11/13/13	to	12/13/13		12/16/13	to	12/20/13	900	700	5451	-263	-263	8	2	40	1	0.2
11/20/13	to	12/20/13		12/23/13	to	12/27/13	538	438	5889	-113	-113	4	1	80	0.6	1
11/27/13	to	12/27/13		12/30/13	to	01/03/14	-50	-175	5714	-325	-325	5	1	60	0.8	1
12/04/13	to	01/03/14		01/06/14	to	01/10/14	-625	-825	4889	-563	-988	8	2	80	0.2	0.6
12/11/13	to	01/10/14		01/13/14	to	01/17/14	-188	-388	4501	-563	-563	8	3	60	0.8	0.8
12/18/13	to	01/17/14		01/20/14	to	01/24/14	1288	1188	5689	-38	-38	4	1	20	1	0.2
12/25/13	to	01/24/14		01/27/14	to	01/31/14	863	288	5977	-325	-388	23	3	20	0.4	0.2

In-Sample Dates				Out_Of_Sample Dates			osnp	NOnp$25	NetEq	ollt	odd	ont	degree	NDays	pctup	pctdn
01/01/14	to	01/31/14		02/03/14	to	02/07/14	1613	1438	7415	-638	-888	7	3	80	0.8	1
01/08/14	to	02/07/14		02/10/14	to	02/14/14	638	463	7878	-450	-1013	7	1	80	0.6	0.4
01/15/14	to	02/14/14		02/17/14	to	02/21/14	-1250	-1400	6478	-738	-1363	6	1	60	0.4	1
01/22/14	to	02/21/14		02/24/14	to	02/28/14	-575	-775	5703	-563	-938	8	3	80	0.2	0.8
01/29/14	to	02/28/14		03/03/14	to	03/07/14	13	-112	5591	-188	-188	5	2	20	0.2	0.8
02/05/14	to	03/07/14		03/10/14	to	03/14/14	450	300	5891	-500	-825	6	1	40	0.8	0.2
02/12/14	to	03/14/14		03/17/14	to	03/21/14	-888	-1088	4803	-750	-1500	8	1	40	1	0.2
02/19/14	to	03/21/14		03/24/14	to	03/28/14	1775	1625	6428	-75	-75	6	3	20	1	0.2
02/26/14	to	03/28/14		03/31/14	to	04/04/14	113	-87	6341	-250	-925	8	1	20	0.8	0.2
03/05/14	to	04/04/14		04/07/14	to	04/11/14	2400	2250	8591	-600	-600	6	2	20	1	0.8
03/12/14	to	04/11/14		04/14/14	to	04/18/14	-1838	-2038	6553	-813	-2975	8	3	60	1	0.8
03/19/14	to	04/18/14		04/21/14	to	04/25/14	-138	-288	6265	-350	-400	6	1	20	1	0.2
03/26/14	to	04/25/14		04/28/14	to	05/02/14	-300	-525	5740	-350	-663	9	1	20	0.8	0.4
04/02/14	to	05/02/14		05/05/14	to	05/09/14	-200	-425	5315	-700	-1088	9	2	20	0.2	1
04/09/14	to	05/09/14		05/12/14	to	05/16/14	800	650	5965	-313	-325	6	2	20	0.2	1
04/16/14	to	05/16/14		05/19/14	to	05/23/14	363	213	6178	-225	-225	6	3	40	1	0.2
04/23/14	to	05/23/14		05/26/14	to	05/30/14	688	588	6766	-25	-25	4	3	40	1	0.4
04/30/14	to	05/30/14		06/02/14	to	06/06/14	1175	1050	7816	0	0	5	3	80	1	0.4
05/07/14	to	06/06/14		06/09/14	to	06/13/14	-38	-188	7628	-263	-450	6	1	60	1	0.8
05/14/14	to	06/13/14		06/16/14	to	06/20/14	288	163	7791	-213	-213	5	1	80	1	0.8
05/21/14	to	06/20/14		06/23/14	to	06/27/14	-88	-288	7503	-363	-488	8	1	80	0.2	0.6
05/28/14	to	06/27/14		06/30/14	to	07/04/14	550	450	7953	-25	-25	4	3	40	0.2	0.4
06/04/14	to	07/04/14		07/07/14	to	07/11/14	950	775	8728	-88	-88	7	2	20	0.2	0.6
06/11/14	to	07/11/14		07/14/14	to	07/18/14	-138	-488	8240	-363	-1350	14	2	20	0.2	0.4
06/18/14	to	07/18/14		07/21/14	to	07/25/14	38	-62	8178	-75	-138	4	3	20	0.6	0.2
06/25/14	to	07/25/14		07/28/14	to	08/01/14	1275	1000	9178	-350	-388	11	2	20	0.6	0.2
07/02/14	to	08/01/14		08/04/14	to	08/08/14	238	63	9241	-750	-750	7	1	80	1	0.2
07/09/14	to	08/08/14		08/11/14	to	08/15/14	-225	-400	8841	-338	-338	7	3	40	1	0.4
07/16/14	to	08/15/14		08/18/14	to	08/22/14	750	625	9466	0	0	5	1	60	0.8	0.2
07/23/14	to	08/22/14		08/25/14	to	08/29/14	-413	-563	8903	-175	-500	6	3	20	0.6	0.4
07/30/14	to	08/29/14		09/01/14	to	09/05/14	-400	-500	8403	-550	-875	4	1	60	0.2	1
08/06/14	to	09/05/14		09/08/14	to	09/12/14	-713	-838	7565	-463	-725	5	2	80	0.2	1
08/13/14	to	09/12/14		09/15/14	to	09/19/14	-763	-1088	6477	-338	-1138	13	3	40	0.4	0.2
08/20/14	to	09/19/14		09/22/14	to	09/26/14	1963	1788	8265	-338	-338	7	1	40	0.4	0.6
08/27/14	to	09/26/14		09/29/14	to	10/03/14	-663	-888	7377	-1075	-1550	9	3	80	0.4	1
09/03/14	to	10/03/14		10/06/14	to	10/10/14	188	-137	7240	-800	-1825	13	3	20	0.6	0.8
09/10/14	to	10/10/14		10/13/14	to	10/17/14	1550	1275	8515	-800	-1088	11	1	60	1	0.4
09/17/14	to	10/17/14		10/20/14	to	10/24/14	1738	1538	10053	-475	-475	8	1	60	0.4	0.4
09/24/14	to	10/24/14		10/27/14	to	10/31/14	1425	1250	11303	-913	-1088	7	3	40	0.2	1
10/01/14	to	10/31/14		11/03/14	to	11/07/14	450	275	11578	-563	-775	7	3	40	0.2	1
10/08/14	to	11/07/14		11/10/14	to	11/14/14	488	363	11941	-200	-200	5	3	60	0.4	1
10/15/14	to	11/14/14		11/17/14	to	11/21/14	-50	-200	11741	-663	-963	6	3	40	0.8	0.8
10/22/14	to	11/21/14		11/24/14	to	11/28/14	-88	-188	11553	-175	-200	4	3	20	0.8	0.2
10/29/14	to	11/28/14		12/01/14	to	12/05/14	-700	-825	10728	-388	-500	5	3	80	1	0.2
11/05/14	to	12/05/14		12/08/14	to	12/12/14	3663	3463	14191	-375	-513	8	3	40	0.6	0.2
11/12/14	to	12/12/14		12/15/14	to	12/19/14	-350	-725	13466	-875	-2200	15	1	40	0.8	0.4
11/19/14	to	12/19/14		12/22/14	to	12/26/14	-213	-313	13153	-163	-413	4	1	80	0.8	0.6
11/26/14	to	12/26/14		12/29/14	to	01/02/15	675	550	13703	-388	-713	5	2	80	1	0.8
12/03/14	to	01/02/15		01/05/15	to	01/09/15	713	538	14241	-713	-713	7	2	80	1	1
12/10/14	to	01/09/15		01/12/15	to	01/16/15	-1550	-1775	12466	-1038	-1913	9	3	60	1	1
12/17/14	to	01/16/15		01/19/15	to	01/23/15	763	563	13029	-350	-350	8	1	60	0.6	0.4
12/24/14	to	01/23/15		01/26/15	to	01/30/15	-1050	-1400	11629	-838	-2088	14	1	60	0.2	0.8
12/31/14	to	01/30/15		02/02/15	to	02/06/15	888	713	12342	-650	-788	7	1	80	0.2	0.8
01/07/15	to	02/06/15		02/09/15	to	02/13/15	1175	1025	13367	-88	-125	6	2	20	0.2	1

In-Sample Dates				Out_Of_Sample Dates			osnp	NOnp$25	NetEq	ollt	odd	ont	degree	NDays	pctup	pctdn
01/14/15	to	02/13/15		02/16/15	to	02/20/15	1438	1338	14705	0	0	4	3	20	0.6	0.8
01/21/15	to	02/20/15		02/23/15	to	02/27/15	675	550	15255	-188	-188	5	3	40	0.8	1
01/28/15	to	02/27/15		03/02/15	to	03/06/15	625	500	15755	-300	-425	5	1	80	0.6	1
02/04/15	to	03/06/15		03/09/15	to	03/13/15	325	175	15930	-625	-950	6	1	20	0.8	1
02/11/15	to	03/13/15		03/16/15	to	03/20/15	-350	-575	15355	-1025	-1675	9	2	60	0.8	0.2
02/18/15	to	03/20/15		03/23/15	to	03/27/15	1963	1788	17143	-388	-388	7	1	60	0.4	0.2
02/25/15	to	03/27/15		03/30/15	to	04/03/15	-150	-275	16868	-488	-763	5	2	20	0.8	0.4
03/04/15	to	04/03/15		04/06/15	to	04/10/15	975	850	17718	-400	-600	5	2	20	0.4	0.8
03/11/15	to	04/10/15		04/13/15	to	04/17/15	-413	-563	17155	-575	-850	6	1	80	0.6	0.6
03/18/15	to	04/17/15		04/20/15	to	04/24/15	713	588	17743	-225	-225	5	3	40	0.2	0.8
03/25/15	to	04/24/15		04/27/15	to	05/01/15	-425	-575	17168	-775	-1450	6	3	40	1	0.8

Figure 1 Graph of Net Equity Curve Applying the Walk Forward Filter Each Week On ES 1min Bar Prices 6/8/12 – 5/1/15

Note: The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity. The brown line is the equity curve with commissions and the green dots are the new highs in net equity.

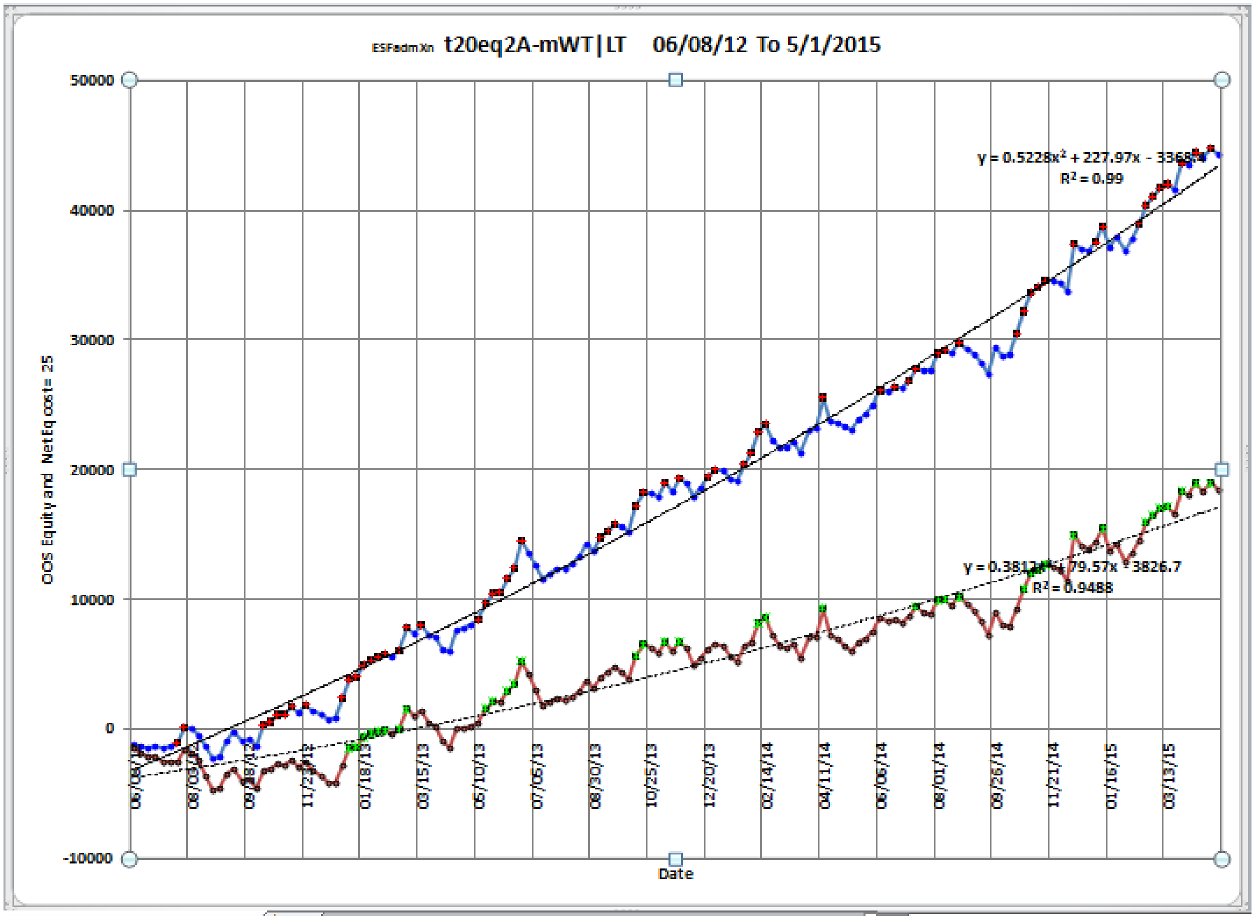

Figure 2 Walk Forward Out-Of-Sample Performance for ES Fading Memory Polynomial Velocity System
1 minute bar chart of ES from 11/7/13-11/7/2013

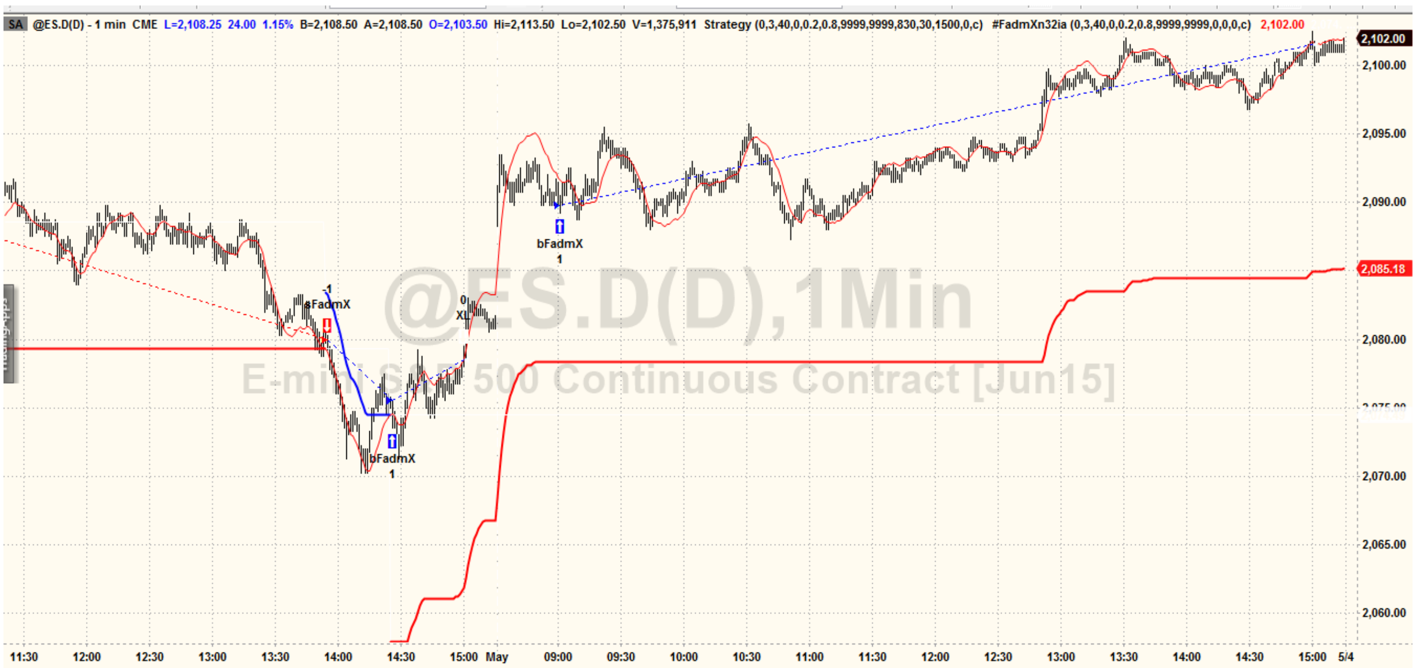

Figure 3_ Partial output of the Walk Forward Metric Performance Explorer (WFME)
ES1 min bars Nth Order Fading Memory System

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	ESFadmXn	s06/08/12	e05/01/15	#152	AnyTnp				a(15313)	s9896	f1922										c=$ 25	
2	Filter-Metric	tOnp	aOnp	aOTrd	aO#T	B0	%P	t	std	LLp	eqDD	lr	#	eqTrn	eqV^3	eqR2	Dev^2	Blw	BE	eff	tOnpNet	Prob
3	t20eq2A-mLTr	47889	315	42.8	7.4	3.3	61	4.41	880	-2100	-3177	5	152	331	680	95	3337	11	31.2	1.09	19914	1.86E-04
4	t20eq2A-mWT|LT	44344	292	42.7	6.8	2.2	59	4.21	854	-1838	-2976	4	152	308	449	99	1633	10	34.3	1.08	18394	3.30E-04
5	t10eq2V-mDev	41692	274	38.9	7.1	3.5	58	3.47	976	-2425	-5138	5	152	255	935	88	4147	19	50.6	0.58	14892	1.14E-03
6	t20llt-eq2A	53434	352	34.4	10.2	4.7	61	4.23	1025	-1875	-5089	6	152	365	982	86	6525	30	34	0.91	14559	1.27E-03
7	t10eq2A-mWT|LT	39470	261	39	6.7	0.5	62	3.84	836	-1863	-2851	4	151	246	481	98	1659	9	40.9	1.39	14170	1.45E-03

The WFME Filter Output Columns are defined as follows:

Row 1 ESFadmXn is the strategy abbreviation, First OOS Week End Date(6/8/12), Last OOS Week End Date(5/1/15), **Number of weeks**(#152) **a**=average of bootstrap random picks. **s**= standard deviation of bootstrap random picks. **f**=number of different filters examined. **c**= slippage and round trip trade cost(c=$30).

Filter = The filter that was run. Row 3 filter **t20eq2A-mWT|LT**

The **t20eq2A-mWT|LT** filter produced the following average 152 week statistics on row 3.

tOnp = Total out-of-sample(oos) net profit for these 152 weeks.

aOsp = Average oos net profit for the 152 weeks

aOTrd = Average oos profit per trade

aO#T = Average number of oos trades per week

B0 = The 152 week trend of the out-of-sample weekly profits

%P = The percentage of oos weeks that were profitable

t = The student t statistic for the 152 weekly oos profits. The higher the t statistic the higher the probability that this result was not due to pure chance

std = The standard deviation of the 152 weekly oos profits

LLp = The largest losing oos period(week)

eqDD = The oos equity drawdown

lr = The largest number of losing oos weeks in a row

= The number of weeks this filter produced a weekly result. Note for some weeks there can be no strategy inputs that satisfy a given filter's criteria.

eqTrn = The straight line trend of the oos gross profit equity curve in $/week.

eqV^3 = The ending velocity of 3rd order polynomial that is fit to the equity curve

eqR2 = The correlation coefficient(r^2) of a straight line fit to the equity curve

Dev^2 = A measure of equity curve smoothness. The square root of the average [(equity curve minus a straight line)^2]

Blw = The maximum number of weeks the oos equity curve failed to make a new high.

BE = Break even weeks. Assuming the average and standard deviation are from a normal distribution, this is the number of weeks you would have to trade to have a 98% probability that your oos equity is above zero.

eff = Efficiency. The average daily out-of-sample profit divided by the average daily in-sample profit.

tOnpNet = Total out-of-sample net profit(tOnpNet) minus the total trade cost.
tOnpNet=tOnp **–** (Number of trade weeks)*aOnT*Cost.

Prob = the probability that the filter's tOnpNet was due to pure chance.

Appendix 1: n^{th} Order Fading Memory Polynomial Next Bar's Forecast Math

The N^{th} Order Fading Memory Polynomial

This is a mathematical technique that fits a n^{th} order polynomial to the last ***T*** price bars but calculates the coefficients of the polynomial such that the error between the current n^{th} order polynomial and the current bar is weighted much higher than the error between the price T bars ago and the value of the n^{th} order polynomial T bars ago. As an example, if the latest price is at time t and the price made a turn at time bar t-10 , then we do not want prices prior to t-10 affecting the current polynomial fit as much. As will be shown the most familiar case of this fading memory technique is the exponential moving average. The fading memory technique is in contrast to the Least Squares Polynomial fit, which weights all past errors between the polynomial and the price bar equally.

Consider a time series x(t) where t is an integer value (a price bar number) like the number of days or minutes, etc from some starting time. Suppose we want to find at some given time some *n*th-degree polynomial that fits the data well at current and recent prices but ignores the fit as we move into the distant past. One way to construct this type of fit would be to weight the past data with a number that got smaller and smaller the further back in time we went. If we let the polynomial function be represented by the symbol **p(t-τ)** where **p(t-0)** is the current value of the polynomial, **p(t-1)** is the previous value of the polynomial, etc., then an error function can be formed that consists of the weighted sum of the squared difference between the price series **x(t-τ)** and the polynomial p(t-τ) given by

$$\textbf{error} = \sum \beta^{\tau}(x(t-\tau) - p(t-\tau))^2 \quad \tau=0 \text{ to } \infty \qquad (1)$$

where $0 < \beta < 1$ **and** β^{τ} is much much less than **1** for large **τ.**

It turns out that if we let the n^{th} degree polynomial **p(t-τ)** be constructed as a linear combination of orthogonal polynomials called Meixner polynomials then minimizing the error with respect to the coefficients of the orthogonal polynomials yields the best estimate of **x(t-τ)** as **Pf(t-τ)** and given by the equation

$$x_{est}(t-\tau) = (1-\beta)\sum_{k=0}^{n} \beta^k b_{k,t}\Phi_k(t)|_{\tau} \qquad (2)$$

Where

$$\Phi_n(t) = \sum_{k=0}^{n} \binom{n}{k}\binom{t}{k} z^k$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$b_{j,t} = \sum_{k=0}^{\infty} \beta^k \Phi_j(k) x(t-k)$$

$$\mathbf{z = 1 - 1/\beta}$$

where n is the polynomial degree, $\Phi_k(\tau)$ are the Meixner polynomials of degree k (k=0 to n), and $b_k(t)$ are the coefficients that minimize the error of equation (1). Generally the summation for $b_j(t)$ can be terminated when β^k <<1.

Appendix 1: n^{th} Order Fading Memory Polynomial Next Bar's Forecast Math

For the exact mathematical solutions that produce equation (2) and the mathematical descriptions of the Meixner polynomials refer to Reference 1.

To yield the 1 day ahead prediction the above equation becomes;

$$Pf(t+1)=(1-\beta)\sum \beta^k b_{k,t}\Phi_k(-1) \quad k=0 \text{ to } n \qquad (3)$$

After some algebraic manipulation with the Meixner polynomials the $b_{k,t}$ coefficients satisfy the following recursive relationship.(see Reference 1)

$$b_{k,t}=\beta\, b_{k,t-1} +b_{k-1,t}-b_{k-1,t-1}$$

One case is of immediate interest where the polynomial is a constant, that is n=0.

For this case the solution to equation (3) can be found after some algebraic manipulation to be:

$$X0_{est}=\beta^*X0_{est}[1] +(1-\beta)^*x(t) \qquad (4)$$

Where $X0_{est}[1]$ is the previous estimated value, **x(t)** is the current bar's price and where the 0 in $X0_{est}$ indicates that we are estimating a polynomial of degree 0 or simply a constant. If a change of variables is made letting $\alpha=(1-\beta)$ then equation (4) becomes:

$$X0_{est}=(1-\alpha)^*X0_{est}[1] +\alpha^*x(t) \qquad (5)$$

This is the familiar formula for the exponential moving average.

Higher orders of n don't yield such compact solutions as the case where n=0 .equations

$$P_F(T+1) = (1-\beta)^*[b_{0,t}\, \phi_{0|t=-1} + \beta b_{1,t}{}^*\phi_{1|t=-1} + \beta^2 b_{2,t}{}^*\phi_{2|t=-1} +\ldots+ \beta^n b_{n,t}{}^*\phi_{n|t=-1}]$$

$$\text{Velocity} = (dP_F/dt)_{(T=-1)} = (1-\beta)[\beta b_{1,t}\, {}^*(d\phi_1/dt)_{|t=-1} + \beta^2 b_{2,t}\, {}^*(d\phi_2/dt)_{|t=-1} +\ldots+ \beta^n b_{n,t}\, {}^*(d\phi_n/dt)_{|t=-1)}]$$

$$\text{Accel} = (d^2P_F/d^2t)_{(T=-1)} = (1-\beta)[\beta^2 b_{2,t}\, {}^*(d^2\phi_2/d^2t)_{|t=-1} + \beta^2 b_{2,t}\, {}^*(d^2\phi_3/d^2t)_{|t=-1} +\ldots+ \beta^n b_{n,t}\, {}^*(d^2\phi_n/d^2t)_{|t=-1}]$$

The n^{th} Order Fading Memory Forecast Next Bar's Price System Defined

The least squares forecast is constructed by solving for the coefficients $b_0, b_1, b_2, \ , \ , b_n$ recursively at each bar using the last **T** bars of closing prices and the Discrete Orthogonal Meixner Polynomial equations above. Then $P_F(T+1)$ is constructed from the equation above and plotted under the price chart. In general what we will be doing is following the plotted curve of P_F which is calculated at each bar from the previous T bars. When the curve increases by a percentage amount ***pctup*** from the previous prior low of the curve we will go long. When the curve falls by the percentage amount ***pctdn*** from the previous prior high of the curve we will go short

Buy Rule:

- **IF** P_f has moved up by more than the percentage amount of ***pctup*** from the lowest low recorded in P_f while short then buy at the market.

Sell Rule:

- **IF P_f** has moved down by more than the percentage amount ***pctdn*** from the highest high recorded in **P_f** while long then sell at the market.

The n^{th} Order Fading Memory Forecast Next Bar's Velocity System Defined

The least squares forecast is constructed by solving for the coefficients $b_0, b_1, b_2, \ , \ , b_n$ recursively at each bar using the last **T** bars of closing prices and the Discrete Orthogonal Meixner Polynomial equations above. Then **Velocity** = $dP_F(T+1)/dt$ is constructed from the velocity equation above and plotted under the price chart. In general what we will be doing is following the plotted curve of **Velocity** which is calculated at each bar from the previous T bars. When the velocity is greater than a threshold amount ***pctup*** we will go long. When the velocity is less than a threshold amount ***-pctdn*** we will go short.

Buy Rule:
IF Velocity is greater than the threshold amount ***pctup*** then buy at the market.

Sell Rule:
IF Velocity is less than the threshold amount ***-pctdn*** then sell at the market.

The n^{th} Order Fading Memory Forecast Next Bar's Acceleration System Defined

The least squares forecast is constructed by solving for the coefficients $b_0, b_1, b_2, \ , \ , b_n$ recursively at each bar using the last **T** bars of closing prices and the Discrete Orthogonal Meixner Polynomial equations above.. Then **Acceleration** = $d^2P_F(T+1)/d^2t$ is constructed from the acceleration equation above and plotted under the price chart. In general what we will be doing is following the plotted curve of **Acceleration** which is calculated at each bar from the previous T bars. When the acceleration is greater than a threshold amount ***aup*** we will go long. When the velocity is less than a threshold amount ***-adn*** we will go short.

Buy Rule:
IF acceleration is greater than the threshold amount ***aup*** then buy at the market.

Sell Rule:
IF acceleration is less than the threshold amount ***-adn*** then sell at the market.

References

1. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.

Appendix 2 Power Walk Forward Optimizer Performance Metrics

The PWFO Metric Performance Statistics Defined.

Shown below is the Excel spreadsheet of a PWFO file for the first 13 cases of an optimization run using the Goertzel Discrete Fourier strategy and The Power Walk Forward Optimizer. This example is not meant to show a profitable strategy but only the definitions of the PWFO columns. This strategy was run on the Russell 2000 futures five minute bars for a in-sample period of 30 calendar days from 7/26/2012 to 8/24/2012 and an out-of-sample period of 7 calendar days from 8/25/2012 to 8/31/2012. The first row of the spreadsheet shows the In-Sample(Test) dates and the out-of-sample dates and the “stub” PWFO input. In this case the In-Sample(Test) dates were from 1120726 to 1120824. The out-of-sample dates were from the next trading day after the Friday of 1120824 which was Monday 1120827 to Friday, 1120831. The 112MMDD designation is TradeStation’s compressed date format where 112 stands for the year 2012. 99 would stand for the year 1999 and 104 would stand for the year 2004. Below follows the definition of columns **A** through **AR** of the spreadsheet output.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
1	1120726	1120824	1120831	TF5GZ																			
2	ncy	pup	pdn	fdy	xop	xt	tnp	mTrd	nT	%P	PF	std	t	mLBr	tLBr	mWBr	tWBr	mWB|LB	tWB|LB	m(RU-p)	m(p-RD)	wr	lr
3	1	1	1	1	0	0	1150	-260	22	27	1.13	1041	0.236	65	1086	89	690	1.369	0.635	700	180	2	9
4	3	1	1	1	0	0	3850	-80	26	38	1.57	787	0.96	45	966	71	813	1.578	0.842	500	270	2	6
5	5	1	1	1	0	0	5070	80	32	56	1.82	713	1.256	35	670	60	1109	1.714	1.655	400	290	6	4
6	10	1	1	1	0	0	3270	-50	59	47	1.31	590	0.721	17	669	32	1178	1.882	1.761	260	150	7	6
7	1	1.25	1	1	0	0	2350	-200	22	32	1.3	1025	0.489	69	1016	92	760	1.333	0.748	600	180	2	7
8	3	1.25	1	1	0	0	4010	-60	26	46	1.66	810	0.971	40	798	70	982	1.75	1.231	400	290	2	3
9	5	1.25	1	1	0	0	8230	170	28	61	2.85	767	2.028	42	617	63	1066	1.5	1.728	360	440	6	4
10	10	1.25	1	1	0	0	3070	20	54	52	1.3	589	0.71	18	617	38	1216	2.111	1.971	310	150	7	4
11	1	1.5	1	1	0	0	170	-270	22	36	0.99	1033	0.035	58	950	75	828	1.293	0.872	580	180	2	5
12	3	1.5	1	1	0	0	4990	50	24	54	1.84	837	1.217	45	698	73	1081	1.622	1.549	390	410	2	2
13	5	1.5	1	1	0	0	9730	170	28	64	3.43	814	2.258	44	589	63	1091	1.432	1.852	350	390	6	3

X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR
1																				
2	mWTrd	mLTrd	mWT|LT	dd	llt	eqTrn	eqR2	mDev	eq2b1	eq2V	eq2A	eq2R2	mKr	e-3	eq10	osnp	odd	ollt	ont	aoTrd
3	1360	-500	2.72	-4670	-1220	-142	-53	588	723.12	-758.8	-33.68	-7.8	-24.14	180	-11.3	-260	-600	-370	5	-52
4	700	-260	2.692	-2280	-1240	57.3	20.3	721.8	65.09	104.9	0.76	16.4	7.94	780	3.3	260	-850	-450	8	32.5
5	430	-340	1.265	-1840	-1360	140.8	70.9	678.6	259.98	86.5	-2.71	62	20.74	-1150	6.1	340	-1010	-460	9	37.8
6	210	-210	1	-2340	-1530	94.2	81.7	533.7	-38.48	193.6	1.97	76.5	17.66	-560	6.7	460	-740	-450	12	38.3
7	1370	-480	2.854	-4330	-1070	-107.5	-39.1	593.1	769.2	-737.9	-34.25	-33.5	-18.13	740	-10.5	-320	-620	-380	5	-64
8	560	-370	1.514	-2480	-1450	59.8	21.4	700.7	151.8	47.3	-2.01	2.9	8.54	800	2.9	40	-950	-490	8	5
9	520	-390	1.333	-1570	-1210	288.6	86.8	533.6	580.44	31.4	-9.8	90.3	54.09	-1050	7.9	200	-890	-480	8	25
10	190	-330	0.576	-1860	-1290	96.6	85.2	410	-2.48	161.7	1.52	78.4	23.55	-320	6.1	-800	-1370	-450	13	-61.5
11	940	-570	1.649	-5740	-1470	-235.8	-77.9	618.6	649	-828.6	-33.58	-20.7	-38.12	270	-13.6	-100	-680	-410	5	-20
12	630	-470	1.34	-2610	-1540	131.8	49.3	649.1	327.46	-30.3	-7.45	51.5	20.3	640	2.5	60	-970	-500	8	7.5
13	530	-330	1.606	-1160	-1150	336.7	88.9	457.3	624.98	73.2	-9.85	92.5	73.63	-680	9.5	60	-890	-460	8	7.5

Columns A,B,C. These are the optimization run input parameters for each case.

The Columns below are the performance statistics for the In-Sample(Test) data on each case (row).

Column	Symbol	Column Definition For The In-Sample(Test) Section:
A - F		Input Parameters.
G	tnp	Total Net profits
H	mTrd	The median of all trades in the In-Sample section
I	nT	# of Trades in In-Sample(Test) Section
J	%P	% Profitable Trades in In-Sample(Test) Section
K	PF	Profit Factor
L	std	Standard Deviation of Trades
M	t	Student t Statistic. Used to determine the probability that the average trade Profit >0

Appendix 2 Power Walk Forward Optimizer Performance Metrics

Column	Symbol	Column Definition For The In-Sample(Test) Section:
N	mLBr	The Median of Bars in Losing Trades
O	tLBr	Total Losing Bars in Losing Trades
P	mWBr	The Median of Winning Trades Bars
Q	tWBr	Total Winning Bars in Winning Trades
R	mWb/mLb	Ratio of Median Winning Bars to Median Losing Bars
S	tWb/tLb	Ratio of Total Winning Bars to Total Losing Bars
T	eq2A	Median of all Trades(Maximum Trade Run-up – Final Trade Profit)
U	m(p-rd)	Median of all Trades(Final Trade Profit - Maximum Trade Run-down)
V	wr	Max consecutive winners in a row
W	lr	Max consecutive losers in a row
Y	mWTr	Median of the winning trades
Z	mWT|LT	Median of the losing trades
AA	mWT/LT	Ratio of Median Winning Trades to Median Losing Trades
AB	dd	In-Sample(Test) Section Drawdown
AC	llt	Largest losing trade in In-Sample(Test) Section
AD	eqTrn	Slope of In-Sample Trade Equity Trend Line
AE	eqR2	Trade Equity Trend Line Coefficient Of Correlation r^2
AF	mDev	Median of the absolute deviations of equity from straight line fit to equity curve
AG	eq2b1	Slope Of Equity 2nd Order Polynomial Line Where Equity 2nd Order Line $=b_0 + b_1 * t + b_2 * t^2$
AH	eq2V	Velocity Of Equity 2nd Order Polynomial Line evaluated on last trade i=nT.
AI	eq2A	Acceleration of Equity 2nd Order Polynomial Line evaluated on last trade
AJ	eq2R2	Equity 2nd Order Polynomial Line Coefficient Of Correlation r^2
AK	mKr	Modified k-ratio = eqTrn/mDev.
AL	e-3	End Equity minus Equity 3 Trades before
AM	eq10	Projected Equity 10 Trades in Future Using 2nd Order Polynomial Line/1000.
	um	User Performance Metric
		Column Definition For Out-Of-Sample Section:
AN	osnp	Out-Of-Sample(OOS) Net Profit
AO	odd	OOS drawdown
AP	ollt	OOS largest losing trade
AQ	onT	OOS # of trades
AR	aoTr	Average OOS trade profit.

Explanation of The logic Behind The New Performance Statistics.

A number of performance statistics are new. This section will explain the logic behind the selection of these new statistics.

- **std - Trade Standard Deviation.**
  This is the standard deviation of the trade profits/losses in dollars.
- **t -** Student t-statistic. Used to determine the probability that the Average Trade Profit (tnp/nT)<>0 . Statistically, high values of t indicate that there is a very small probability that the sample average trade profit on the spreadsheet is <= zero and a random number. In real life, I find that if I screen out the top 5% of t values in Excel, I screen out a lot of curve fit input parameters. Same goes for mkr.
- **mLBr - Median Of The Trade Losing Bars.**
  Each losing trade takes a certain number of bars. If we order the number of bars each losing trade takes then the median of all the losing trade bars is a robust statistic. We take the median of the losing trades bars to minimize the effect of large and small losing trade bars that may be outliers that distort this statistic.
- **mWBr -Median Of The Trade Winning Bars**
  Each winning trade takes a certain number of bars. If we order the number of bars each winning trade takes then the median of all the winning trade bars is a robust statistic. We take the median of the winning trades bars to minimize the effect of large and small winning trade bars that may be outliers that that distort this statistic.
- **eq2A – Median of All Trades(Maximum Trade Runup Minus Final Trade Profit)**
  This statistic measures the difference between the maximum profit (trade runup) of each trade and the final profit of the trade. mru is the median of this difference for all trades for the given input variables. The closer the final trade profit is to the maximum trade profit, the better the performance of the input variable. Thus we would want the median to be as small as possible. We use the median for this statistic, because we do not want the statistic distorted by a few outlier trades
- **m(p-rd) – Median of All Trades(Final Trade Profit Minus Maximum rundown of Trade).**
  This statistic measures the difference between the final profit of each trade and the maximum trade loss (rundown) of the trade. The farther the final trade profit is from the maximum trade drawdown, the better the performance of the input variable. Thus, we would want the median to be as large as possible. We use the median for this statistic, because we do not want the statistic distorted by a few outlier trades
- **mWTr- Median Of The Winning Trades.**
  This is the median of the winning trade profits. We take the median of the winning trades to minimize the effect of large winning trades that may be outliers that are not repeatable.
- **mWT|LT -Median Of The Losing Trades**
  This is the median of the losing trade losses. We take the median of the losing trades to minimize the effect of large losing trades that may be outliers that are not repeatable.
- **mWT/LT – Median of Winning Trades divided by Median of Losing Trades|.**
  This is the ratio of mWTr to absolute value of mWT|LT. A high ratio indicates robustness indicating that the statistic of the median of the winning trades to losing trades is not caused by outliers.
- **eqTrn – Slope Of Trade Equity Regression Line.**
  The equity curve is fitted by a straight line where $Equity_{est} = a_0+b_0*t$. $Equity_{est}$ is the straight line estimate of the Equity curve and **t** is the trade number. **b0** is the slope of the straight line which we designate as eqTrn The dimensions of **eqTrn** are dollars per trade.
- **eqR2 - Trade Equity Regression Trend Line Coefficient Of Correlation r2**
  r2 is a measure of how well a straight line fits the equity curve. R2 goes from 0 to 100. An R2 of 100 means the equity curve fits a straight line exactly. In general, a high value of r2 is desirable because it indicates trade profit consistency.
- **mDev - Median Of The Absolute Values Of (The Equity At Each Trade Minus The Equity Regression Trend Line)**
  Dev is the median of all the absolute values of the deviations of the equity curve from the Equity straight line regression. This measure is similar to the standard deviation. However, the standard deviation squares

each error(deviation of the equity curve from the Equity straight line regression) in it's sum it weights large deviations much more strongly and is highly distorted by outliers. I prefer to the robust median statistic.

- **mkr - Modified K-Ratio = 100*eqTrn/mDev.**
  Lars Kestner developed the K-ratio in *Technical Analysis of Stocks and Commodities*, March 1996. the K-ratio compares reward to risk. Lars K-ratio used the Standard deviation from the trend line. I felt this gave to high a weight to the large profits and large losses so I modified the K-ratio by using the mD**ev** described above.
- **eq2B1 - Slope Of Equity Curve Least Squares 2nd Order Polynomial Line $\mathbf{Equity_{est}=b_0+ b_1*i +b_2*i^2}$.**
  The equity curve is fitted by a 2nd order polynomial where $Equity_{est} = b_0+b_1*i+b_2*i^2$. $Equity_{est}$ is the estimate of the Equity curve and **i** is the trade number. $\mathbf{b_1}$ is the slope of the parabolic line. The dimensions of $\mathbf{b_1}$ are dollars per trade. Many times the Equity curve doesn't fit a straight line very well. This is especially true if the equity curve is increasing or decreasing faster near the last trades than it was near the first trades. In this case, b1 gives a better representation of the equity curve trend.
- **eq2V** - **Velocity Of Equity Curve Least Squares 2nd Order Polynomial Line.**
  This is a measure of how fast the equity curve is moving on the last trade. If eq2V is negative and eq2B1 is positive this indicates that the equity curve is trending down on the last trades. Since after the last In-Sample(Test) sample trade comes trades in real time, it's important to know what direction the equity curve was headed before you trade.
- **b2 - Acceleration Of Equity Curve Least Squares 2nd Order Polynomial Line Where $\mathbf{Equity_{est}= b_0+ b_1*i +b_2*i^2}$.**
  The equity curve is fitted by a 2nd order polynomial where $Equity_{est} = b_0+ b_1*i+b_2*i^2$. $Equity_{est}$ is the estimate of the Equity curve and **i** is the trade number. **b2** is the acceleration of the parabolic line. The dimensions of **b2** are dollars per trade^2. Many times the Equity curve doesn't fit a straight line very well. This is especially true if the equity curve is increasing or decreasing faster near the last trades than it was near the first trades. In this case, b2 is a measure of how fast the slope of the equity curve is changing. If b2 is positive the equity curve's slope is increasing upward meaning the equity is increasing faster than a straight line. If b2 is negative, the equity curve's slope is decreasing meaning the equity will start decreasing a some point in time if the equity continues to follow this parabolic curve.
- **eq2R2 - Equity Least Squares 2nd Order Polynomial Line Coefficient Of Correlation $\mathbf{r^2}$**
  eq2R2 is a measure of how well a least squares 2nd order polynomial line fits the equity curve. Note: for a 2nd order polynomial eq2R2 can be negative. This is because the coefficient of correlation is for straight lines. 2nd Order and higher polynomials would be considered nonlinear. An r^2 of 100 means the equity curve fits a parabolic exactly. In general, a high value of r^2 is desirable because it indicates trade profit consistency.
- **e-3 – End Equity Minus Equity 3 Trades before.**
  This is another measure of the equity curve at the end. This shows what the equity trend is doing on the last three trades.
- **eq**10 - **Projected Equity 10 Trades in Future Using Curve Least Squares 2nd Order Polynomial Line/1000.**
  This is another measure of the equity curve 10 trades in the future. The projected equity is divided by 1000. This shows what the best estimate of the equity would be if it followed the least squares parabolic line for ten trades into the future. This performance variable can also serve as a curve fit alert. If eq10 is very high, this would indicate that the input parameters for this case have curve fit the noise and will not work well in the future. Using Excel you could exclude the Top 10% of eq10 cases.