

Trading IWM ETF 5min Bars Using the nth Order Fixed Memory Polynomial Velocity Algorithm
Walk Forward in-sample 20 Trading weekdays and out-of-sample 1 Trading weekday.
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In previous working papers at <https://meyersanalytics.com/papers> we showed how the application of a price curve generated by the **Nth Order Fixed Memory Polynomial Velocity** could be used to develop a strategy to buy and sell futures and stocks intraday. The reasoning behind this type of strategy was to only trade when the price trend velocity was above a certain threshold. Many times, prices meander around without any notable trend, and this is considered noise. During these times we do not wish to trade because of the cost of whipsaw losses that would occur from this type of price action. When a price trend finally starts, the velocity of that price trend moves above a minimum threshold noise value. Thus, the velocity strategy would only issue a trade when certain velocity thresholds above “noise” levels are crossed.

The velocity strategy that we will use here to trade the Russel 2000 ETF (**IWM**) is called the nth Order Polynomial Velocity Strategy. The nth Order Adaptive Polynomial Velocity Strategy has several unknown inputs that we must determine before we can use this strategy to trade. These unknown inputs are the polynomial order or degree, the optimum number of lookback prices we need to determine the coefficients of the polynomial and finally the velocity thresholds. Here we will use Walk Forward Optimization and out-of-sample testing to determine the “best” polynomial inputs as well as how these inputs change over time. We will use the nth Order Fixed Memory Polynomial Velocity Strategy to trade the IWM ETF on an intraday basis using 5-min bar price data from 1/2/2022 to 6/30/2023.

The nth Order Fixed Memory Velocity Strategy Defined

The least squares forecast nth order fixed memory polynomial velocity is constructed by solving for the coefficients $\beta_0, \beta_1, \beta_2, \beta_3 \dots \beta_n$ for the discrete orthogonal Legendre polynomials each day using the last **N bars** of closing prices and the equation for β_j shown in the “Math” appendix at the end of this working paper. Then nth Order Fixed Memory Polynomial **Velocity(T+1)** is constructed from the equation shown in the “Math” appendix.

Due to polynomial mathematics, the Velocity of the 2nd, 3rd and 4th order degree polynomial curve changes faster than it’s corresponding first order degree polynomial velocity. Whether higher order polynomial velocities are an advantage or not, will be determined by the computer when we use a walk forward optimization technique described below.

At each bar we calculate the nth order degree (1st through 4th) fixed memory polynomial velocity from the formulas in the “Math” appendix. As will be shown below, walk forward optimization will determine the **degree** for the nth order polynomial velocity, the number of lookback prices, **N**, needed to compute the polynomial coefficients and the threshold amounts **vup** and **vdn**. When the nth order degree velocity is greater than the threshold amount **vup** we will go long. When the velocity is less than the threshold amount **-vdn** we will go short.

Buy Rule:

IF Velocity is greater or equal than the threshold amount **vup and Velocity [1] is less than vup** then buy at the market.

Sell Rule:

IF Velocity is less than or equal than the threshold amount **-vdn and Velocity [1] is greater than -vdn** then sell at the market.

Where Velocity [1] is the velocity on the previous bar.

Intraday Bars Exit Rule:

Close the position at **1555 EST** (No trades will be carried overnight).

Testing The Polynomial Velocity Strategy Using Walk Forward Optimization

There will be four strategy parameters to determine:

1. **degree**, degree=1 for straight line velocity, degree=2 for 2nd order velocity, etc.
2. **N**, is the number of lookback bars of prices to calculate the **velocity**.
3. **vup**, the threshold amount that velocity must be greater than to issue a buy signal.
4. **vdn**, the threshold amount that velocity must be less than to issue a sell signal.

As mentioned, to test this Strategy we will use five-minute bar prices of the Russel 2000 ETF traded on the NYSE and known by the symbol IWM for the 355 trading days from January 2, 2022, to June 30, 2023.

We will test the FixmVn strategy with the above IWM ETF 5 min bars on a **walk forward basis**, where the in-sample (**IS**) will be 20 trading weekdays and the out-of-sample (**OOS**) will be the next trading weekday following as will be described below.

What Is a Walk Forward Optimization with In-Sample Section and Out-Of-Sample Sections?

Whenever we do a TradeStation or Multicharts (TS/MC) optimization on many different strategy inputs, TS/MC generates a report of performance metrics (total net profits, number of losing trades, etc.) vs these different strategy inputs. If the report is sorted on say the total net profits(**tnp**) performance metric column, then the highest **tnp** would correspond to a certain set of inputs. This is called an **in-sample (IS) section**. If we choose a set of strategy inputs from this report based upon some performance metric, we have no idea whether these strategy inputs will produce the same results on future price data or data they have not been tested on. Price data that is not in the in-sample section is defined as **out-of-sample data**. Since the performance metrics generated in the in-sample section are mostly due to "curve fitting" or "data mining" it is important to see how the strategy inputs chosen from the in-sample section perform on out-of-sample data.

What do we mean by "**curve fitting**" or **data mining**? As a simple example, suppose you were taking a subway to work. In the subway car you are in, suppose you counted the number of blond women in that car and suppose the percent of blond women vs all other women hair colors was 80%. Being that you cannot observe what is in the other subway cars, you would assume that all the other subway cars and perhaps all women had the same percentage of blond hair. This observation was due to chance. That is an example of curve fitting. The same goes for combinatorial searches. You are observing results from a finite sample of data without knowing the data outside the sample you examined.

Walk forward analysis attempts to minimize the curve fitting of price noise by using the law of averages from the Central Limit Theorem on the out-of-sample performance. In walk forward analysis the data is broken up into many in-sample and out-of-sample sections. Usually for any strategy, one has some performance metric selection procedure, which we will call a **filter**, used to select the strategy input parameters from the optimization run. For instance, a **filter** example might be all cases that have a profit factor (PF) greater than 1 and less than 3. For the number of cases left, we might select the cases that had the best percent profit. This procedure would leave you with one case in the in-sample section output and its associated strategy input parameters. Now suppose we ran our optimization on each of our many in-sample sections and applied our **filter** to each in-sample section output. We would then use the strategy input parameters found by the **filter** in each in-sample section on the out-of-sample section immediately following that in-sample section. The input parameters found in each in-sample section and applied to each out-of-sample section would produce independent net profits or losses for each of the out-of-sample sections. Using this method, we now have "x" number of independent out-of-sample section profit and losses from our **filter**. If we take the average of these out-of-sample section net profits and losses, then we will have an estimate of how our strategy will perform on average. Due to the Central Limit Theorem, as your

sample size increases, the spurious noise results in the out-of-sample section performance tend to average out to zero in the limit, leaving us with what to expect from our strategy and filter. **Mathematical note: This assumption assumes that the out-of-sample returns are from probability distributions that have a finite variance.**

Why use the walk forward technique? Why not just perform an optimization on the whole price series and choose the input parameters that give the best total net profits or profit factor or some other performance metric? Surely the price noise cancels itself out with such a large number of in-sample prices and trades. Unfortunately, nothing could be farther from the truth! Optimization is a misnomer and should really be called combinatorial search. As stated above, whenever we run a combinatorial search over many different combinations of input parameters on noisy data on a fixed number of prices, **no matter how many**, the best performance parameters found are guaranteed to be due to **“curve fitting”** the noise and signal. The price series that we trade consists of random spurious price movements, which we call noise, and repeatable price patterns (*if they exist*). When we run, for example, 5000 different inputs parameter combinations, the best performance parameters will be from those strategy input variables that are able to produce profits from the price pattern **and** the random spurious movements. While the price patterns will repeat, the same spurious price movements will not. If the spurious price movements that were captured by a certain set of input parameters were a large part of the total net profits, as they are in real intraday price series, then choosing these input parameters will produce losses when traded on future data. These losses occur because the spurious price movements will not be repeated in the same way. This is why strategy optimization or combinatorial searches, also called back testing, with no out-of-sample testing cause losses when traded in real time from something that looked great in the in-sample section.

To gain confidence that our input parameter selection method or filter, using the optimization output of the in-sample data, will produce profits, we must test the input parameters we found in the in-sample section on out-of-sample data. In addition, we must perform the in-sample/out-of-sample analysis many times. Why not just do the out-of-sample analysis once or just 10 times? Well just as in Poker or any card game, where there is considerable variation in luck from hand to hand, walk forward out-of-sample analysis give considerable variation in out-of-sample profit “luck”. That is, by pure chance we may have chosen some input parameter set that did well in the in-sample section data **and** the out-of-sample section data. In order to minimize this type of “luck”, statistically, we must repeat the walk forward out-of-sample (**OOS**) analysis over many (>50) in-sample/out-of-sample sections and take an average over all out-of-sample sections. This average gives us an expected out-of-sample return and a standard deviation of out-of-sample returns which allows us to statistically estimate the expected equity and its range for N out-of-sample periods in the future.

Finding The FixmVn Strategy Parameters Using Walk Forward Optimization

There are four strategy parameters to find, pw , N , vup , vdn .

For the test data we will run the TS or MC optimization engine on **IWM** 5 min price bars from 01/02/2022 to 6/30/2023 with the following optimization ranges for the FixmVn strategy inputs. This will create **355, 20 weekday in-sample periods each followed by a 1 day out-of-sample period** (See Figure 1 for the in-sample/out-of-sample periods). The days are weekdays only. Weekdays where the OOS falls on an exchange holiday or partial days are eliminated. Holidays that fall on a weekday create a 19-day **IS**. All other **IS** periods consist of 20 trading weekdays. The optimization ranges are:

1. **pw=degree from 1 to 3**
2. **N from 4 to 20 in steps of 1.**
3. **vup from 0.25 to 3.5 steps of 0.25**
4. **vdn from 0.25 to 3.5 in steps of 0.25**
5. **Mult = 23.6, iNorm=1 (See Appendix 3, the Normalization Multiplier)**

The above pw , n , vup , vdn will produce 9996 different input combinations or cases of the strategy input parameters for each of the 355 in-sample/out-of-sample files for the 18 months of 5 min bar IWM data.

Finding the Best Set of Strategy Inputs to use with an in-sample Metric Filter.

The PWFO generates a number of performance metrics in the in-sample section. (Please see <http://meyersanalytics.com/Walk-Forward-Optimization.html> for a listing of these performance metrics). The question we are attempting to answer statistically, is which performance metric or combination of performance metrics (which we will call a **filter**) applied to a given set of strategy inputs in the **in-sample** section will produce statistically valid profits in the sum of all out-of-sample sections. In other words, we wish to find the best set of strategy inputs **with a metric filter applied** in each **in-sample** section that gives the “best” total out-of-sample results over all out-of-sample sections. This means if we applied our **metric filter** to the strategy inputs chosen in the in-sample section, we would **only trade using those set of strategy inputs** in the next out-of-sample section if the in-sample **metric filter** satisfied our criteria. **Else no trades would be made** in the next out-of-sample section.

The Walk Forward Strategy – Strategy Inputs with Metric Filters Explorer.

We wish to find **one** set of strategy inputs that we can trade in every out-of-sample section, but we will only trade that set of strategy inputs in the out-of-sample section if and only if they satisfy our in-sample **metric-filter**. Else we will not trade the next out-of-sample section. In this paper the in-sample section is 20 trading days, and the out-of-sample section is the next trading day. After running the PWFO on the in-sample data, we examine the in-sample metric filter that we chose. If the strategy inputs we selected satisfy the in-sample metric filter requirements then we use those strategy inputs to trade the next day. If the strategy inputs do not satisfy the in-sample metric filter, we do not trade the next day.

Let us define the in-sample **metric-filter** we will use here: in-sample Profit Factor ($PF \leq x$) and/or Losers in a row ($lr \leq y$), and/or equity curve straight line correlation coefficient ($r^2(R2) \leq z$). That is **$PF \leq x$ and/or $lr \leq y$ and/or $R2 \leq z$** .

What we are going to do here is look at every combination in the in-sample section of each **strategy input** with **$PF \leq x$ and/or $lr \leq y$ and/or $R2 \leq z$** . This will produce seven **strategy input | metric-filter** combinations:

1. **strategy input | $PF \leq x, lr \leq y, R2 \leq z$ |**
2. **strategy input | $PF \leq x, lr \leq y$ |**
3. **strategy input | $PF \leq x, R2 \leq z$ |**
4. **strategy input | $PF \leq x$ |**
5. **strategy input | $LR \leq y, R2 \leq z$ |**
6. **strategy input | $lr \leq y$ |**
7. **strategy input | $R2 \leq z$ |**
8. **strategy input – we also examine inputs with no filter.**

If the **strategy input | metric-filter** satisfies **the metric-filter** condition in the in-sample section, then we will use those strategy inputs to trade in the out-of-sample section. If not, then there will be no trades in the out-of-sample section.

We will look at all **metric-filter** combinations of **$PF \leq 2$ to 5 step 1, $LR \leq 3, 5$ step 2 and $R2 \leq 60$ to 80 step 10**. We will also look at the strategy input with no metric-filter. With 9996 different strategy input combinations this will give us 444527 **strategy input | metric-filter** combinations. Each one of these 444527-**strategy input | metric-filter** combinations will be applied to each in-sample section and their out-of-sample performance will be tabulated for all 355 PWFO files.

Below is a snippet of the output from a run of all 599759 combinations sorted by **tONP = total OOS net profit for each strategy input | metric-filter** combination. **The column definitions are defined in Figure 3 below**. This example shows a partial output file from the WFINP program run on the PWFO files generated with the FixmVn that was run on 100 shares of IWM ETF 5-minute bars 355 days from 01/02/2022 to 6/30/2023. The in-sample (IS) period is 20 trading weekdays, and the out-of-sample (OOS) period is 1 trading weekday. This strategy traded between 9am to 1600pm Exchange Time (EST).

From this run, we chose the filter on row 4 of the Figure below. That is,

3|13|2|2|0|1555|23.6|pf<2. This is constructed as follows.

For the strategy inputs **3|13|2|2|0|1555|23.6|**only those in-sample sections that have a **pf ≤ 2** is used to trade in the following out-of-sample next trading day section. If the in-sample **pf > 2** then the next trading day out-of-sample section **is not** traded.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1 IWM5mFixmV20x1dxo	s02/01/22	e06/30/23	#355	AnyTnp																					
2 pw[N vup vdn xop xt mult <PF<LR<R2	toGP	TONP	aoGP	aoTr	ao#T	#	std	skew	kur	t	oW oL	%Wtr	%P	LLtr	LLp	eqDD	wpr	lpr	V20	KTau	eqR2	Blw	BE	tkr bl	Prob
3 3 13 2 2 0 1555 23.6 pf<2	12286	10726	68	31.5	2.1	182	227	-0.097	3.22	4.02	1.64	50	64	-408	-645	-1069	10	6	5	92	89	45	92	786	2.68E-11
4 3 13 2 2 0 1555 23.6 pf<4 r2<80	12064	10524	66	31.3	2.1	184	234	-0.071	3.47	3.81	1.64	50	65	-408	-645	-1588	9	5	5	92	94	50	103	635	8.50E-11
5 3 13 2 2 0 1555 23.6 pf<3 r2<80	11905	10405	67	31.7	2.1	179	229	-0.169	3.46	3.89	1.65	50	65	-408	-645	-1588	11	5	7	92	93	50	99	672	4.59E-11
6 3 13 2 2 0 1555 23.6 pf<2 r2<80	11748	10384	73	34.5	2.1	160	228	-0.126	3.37	4.07	1.7	50	66	-408	-645	-989	10	5	6	92	92	50	90	763	4.04E-13
7 3 13 2 2 0 1555 23.6 pf<3	11965	10193	56	27	2.1	212	226	-0.116	3.3	3.64	1.57	49	62	-408	-645	-1588	8	6	7	92	92	47	113	576	2.24E-08
8 3 13 2 2 0 1555 23.6 r2<80	11745	10185	63	30.1	2.1	187	233	-0.05	3.44	3.68	1.64	49	64	-408	-645	-1588	9	5	6	92	93	50	110	569	4.91E-10
9 3 13 2 2 0 1555 23.6 pf<5 r2<80	11745	10185	63	30.1	2.1	187	233	-0.05	3.44	3.68	1.64	49	64	-408	-645	-1588	9	5	6	92	93	50	110	569	4.91E-10
10 3 13 2 2 0 1555 23.6 pf<4	12011	10175	54	26.2	2.1	222	234	-0.131	3.49	3.44	1.54	49	62	-675	-675	-1588	11	6	5	91	93	47	126	495	7.79E-08
11 3 13 1.5 2 0 1555 23.6 pf<3	12213	9909	48	21.2	2.3	252	223	-0.323	4.08	3.45	1.44	50	62	-408	-708	-1314	11	8	10	92	97	70	126	350	2.20E-06
12 3 13 2 2 0 1555 23.6 pf<5	11506	9626	50	24.5	2.1	228	234	-0.112	3.44	3.26	1.53	49	61	-675	-675	-1588	11	6	5	91	92	47	141	414	5.23E-07
13 3 13 1.5 2 0 1555 23.6 pf<4	11914	9510	44	19.8	2.2	268	225	-0.371	4.14	3.24	1.43	50	61	-675	-708	-1314	11	8	10	91	97	70	142	289	1.30E-05
14 3 13 2 2 0 1555 23.6	11366	9482	50	24.1	2.1	229	234	-0.103	3.44	3.21	1.53	49	61	-675	-675	-1588	9	6	5	91	92	47	145	397	7.88E-07
15 3 13 2 2 0 1555 23.6 pf<2 r2<70	10639	9463	76	36.2	2.1	140	210	0.154	3.02	4.27	1.71	51	68	-408	-472	-812	10	3	6	93	94	50	82	915	5.45E-14

Bootstrap Probability of Filter Results.

Using modern "Bootstrap" techniques, we can calculate the probability of obtaining our filter's total out-of-sample **net** profits by chance. Here is how the bootstrap technique is applied. Suppose as an example, we have 500 files of in-sample/out-of-sample data. A mirror random filter is created. Instead of picking an out-of-sample net profit (OSNP) from a filter row as before, the mirror filter picks a **random** row's OSNP in each of the 500 files. We repeat this random picking in each of the 500 files 5000 times. Each of the 5000 mirror filters will choose a random row's OSNP of their own in each of the 500 files. At the end, each of the 5000 mirror filters will have 500 **random** OSNP's picked from the rows of the 500 files. The sum of the 500 random OSNP picks for each mirror filter will generate a random total out-of-sample net profit (toNP) or final random equity. The average and standard deviation of the 5000-mirror filter's different random toNPs will allow us to calculate the chance probability of our above chosen filter's toNP. Thus, given the mirror filter's bootstrap random toNP average and standard deviation, we can calculate the probability of obtaining our chosen filter's toNP by pure chance alone. Figure 3 lists the 5000-mirror filter's bootstrap average for our 355 out-of-sample files of **-\$6.4** with a bootstrap standard deviation of **\$10.0**. (Side Note. The average is the average per out-of-sample period. So, the average for the random selection would be the random toNP/355 and the average for the filter would be the filter toNP/# of OOS periods traded or 10726/1182=58.93). The probability of obtaining our filters average daily net profit of **58.93** is **2.68⁻¹¹** which is **6.5** standard deviations from the bootstrap average. For our filter, in row 4 above, the expected number of cases that we could obtain by pure chance that would match or exceed **\$58.93** is $[1-(1-2.68 \times 10^{-11})^{599759}] \sim 599759 * 2.68 \times 10^{-11} = 0.0000161$ where **599759** is the total number of different filters we looked at in this run. This number is much much less than one, so it is improbable that our result was due to pure chance.

Results

Figure 1 presents a graph of the equity curve generated by using the filter on the 355 days from 2/2/22 to 6/30/23. The equity curves are plotted from Equity and Net Equity columns in Table 1. Plotted on the equity curves is the 2nd Order Polynomial curve. The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity. The brown line is the equity curve with commissions and the green dots are the new highs in net equity. The grey line is the IWM Daily Closing prices superimposed on the Equity Chart.

Figure 2 presents a plot of the FixmVn Strategy buy/sells and the FixmVn Indicator on the IWM 5min bars for 6/22/2023 - 6/30/2023.

Table 1 below presents a table of the 355 in-sample and out-of-sample windows, the **Filter** selected in-sample strategy inputs and the daily out-of-sample profit/loss results using the filter described above.

Discussion of Strategy Performance

In Figure 3, Row 4 of the spreadsheet filter output are some statistics that are of interest for our filter. An interesting statistic is **Blw**. **Blw** is the maximum number of days the OSNP equity curve failed to make a new high. **Blw** is **45** days for this filter. This means that 45 trading days was the longest time that the equity for this strategy failed to make a new equity high. **%Wtr** is the percentage of all OOS trades that were wins or positive. For this filter, the **%Wtr=50%**. **%P** is the % winning oos days, **%P=64%**. The average oos winning trade to the average oos losing trade ratio(**oW|oL**) was **1.64**. **wpr=10** is the maximum number of consecutive winning oos periods(days) in a row and **lpr=6** is the maximum number of consecutive losing oos periods(days) in a row. The Largest losing trade in the whole period was **(\$408)** and the largest losing day was **(\$645)**.

In Figure 1, which presents a graph of the equity curve using the filter on the 355 trading days of out-of-sample data, notice how the equity curve follows the 2nd order polynomial trend line with an R² of 0.973. The R² only dropped to 0.966 for the net equity curve.

Using this filter, the strategy was able to generate \$11309 net equity after commissions of \$0 (many brokers today, 6/30/23, don't charge commissions) and roundtrip slippage of \$4 trading 100 IWM ETF shares for 355 days. This period of time from 3/29/22 to 3/2/23 was a volatile down then up market. Yet the FixmVn strategy was able to adapt quite well. However, from 3/2/23 to 6/30/23 the return was relatively flat.

In observing Table 1 we can see that this strategy and filter made trades from a low of no trades/day to a high of 7 trades/day with an average of 2.1 trades/day on the days it traded. For the no trade days, the strategy **input|filter** in the in-sample section didn't satisfy the metric filter and no trades were made the next trading day. The **input|filter** traded 182 days out of the 355 days or about 51% of the time.

References

1. Efron, B., Tibshirani, R.J., (1993), "An Introduction to the Bootstrap", New York, Chapman & Hall/CRC.
2. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.

Figure 1 Graph of FixmVn Strategy Equity Applying the Walk Forward Filter Each Day on the in-sample section on IWM 5min Bar Prices 01/02/2022 to 7/9/2020

Note: The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity.

The brown line is the equity curve with commissions and the green dots are the new highs in net equity.

The grey line is the IWM Daily Closing prices superimposed on the Equity Chart.

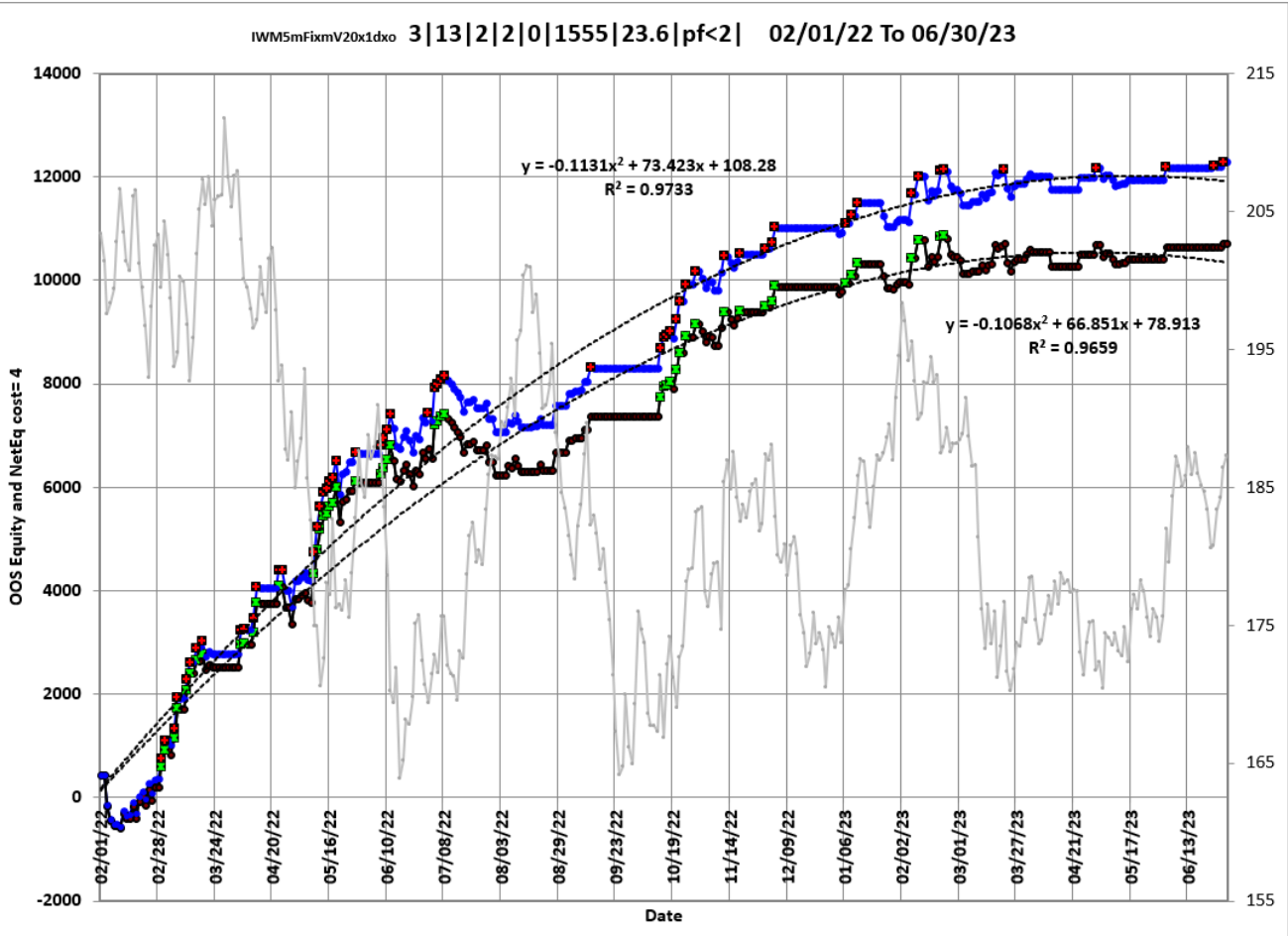


Figure 2 Walk Forward Out-Of-Sample Performance Summary for nth Order Fixed Memory Polynomial Velocity Strategy IWM 5-minute bar chart from 6/22/23 to 6/30/23



Figure 3 Partial output of the Walk Forward Strategy Inputs with Metric Filters (WFINP) IWM ETF 5 min bars Using the FixmVn Strategy

The WFINP Filter Output Columns are defined as follows: OOS=out-of-sample

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	IWM5mFixmV20x1dxo	s02/01/22	e06/30/23	#355	AnyTnp					ISnt2				a(6.4)	s10.0	f599759										
2	pw N vup vdn xop xt mult <PF<LR<R2	toGP	toNP	aoGP	aoTr	ao#T	#	std	skew	kur	t	oW oL	%Wtr	%P	LLtr	LLp	eqDD	wpr	lpr	V20	KTau	eqR2	Blw	BE	tkr bl	Prob
3	3 13 2 2 0 1555 23.6 pf<2	12286	10726	68	31.5	2.1	182	227	-0.097	3.22	4.02	1.64	50	64	-408	-645	-1069	10	6	5	92	89	45	92	786	2.68E-11
4	3 13 2 2 0 1555 23.6 pf<4 r2<80	12064	10524	66	31.3	2.1	184	234	-0.071	3.47	3.81	1.64	50	65	-408	-645	-1588	9	5	5	92	94	50	103	635	8.50E-11
5	3 13 2 2 0 1555 23.6 pf<3 r2<80	11905	10405	67	31.7	2.1	179	229	-0.169	3.46	3.89	1.65	50	65	-408	-645	-1588	11	5	7	92	93	50	99	672	4.59E-11
6	3 13 2 2 0 1555 23.6 pf<2 r2<80	11748	10384	73	34.5	2.1	160	228	-0.126	3.37	4.07	1.7	50	66	-408	-645	-989	10	5	6	92	92	50	90	763	4.04E-13
7	3 13 2 2 0 1555 23.6 pf<3	11965	10193	56	27	2.1	212	226	-0.116	3.3	3.64	1.57	49	62	-408	-645	-1588	8	6	7	92	92	47	113	576	2.24E-08
8	3 13 2 2 0 1555 23.6 r2<80	11745	10185	63	30.1	2.1	187	233	-0.05	3.44	3.68	1.64	49	64	-408	-645	-1588	9	5	6	92	93	50	110	569	4.91E-10
9	3 13 2 2 0 1555 23.6 pf<5 r2<80	11745	10185	63	30.1	2.1	187	233	-0.05	3.44	3.68	1.64	49	64	-408	-645	-1588	9	5	6	92	93	50	110	569	4.91E-10
10	3 13 2 2 0 1555 23.6 pf<4	12011	10175	54	26.2	2.1	222	234	-0.131	3.49	3.44	1.54	49	62	-675	-675	-1588	11	6	5	91	93	47	126	495	7.79E-08
11	3 13 1.5 2 0 1555 23.6 pf<3	12213	9909	48	21.2	2.3	252	223	-0.323	4.08	3.45	1.44	50	62	-408	-708	-1314	11	8	10	92	97	70	126	350	2.20E-06
12	3 13 2 2 0 1555 23.6 pf<5	11506	9626	50	24.5	2.1	228	234	-0.112	3.44	3.26	1.53	49	61	-675	-675	-1588	11	6	5	91	92	47	141	414	5.23E-07
13	3 13 1.5 2 0 1555 23.6 pf<4	11914	9510	44	19.8	2.2	268	225	-0.371	4.14	3.24	1.43	50	61	-675	-708	-1314	11	8	10	91	97	70	142	289	1.30E-05
14	3 13 2 2 0 1555 23.6	11366	9482	50	24.1	2.1	229	234	-0.103	3.44	3.21	1.53	49	61	-675	-675	-1588	9	6	5	91	92	47	145	397	7.88E-07
15	3 13 2 2 0 1555 23.6 pf<2 r2<70	10639	9463	76	36.2	2.1	140	210	0.154	3.02	4.27	1.71	51	68	-408	-472	-812	10	3	6	93	94	50	82	915	5.45E-14

Row 1 IWM5Fixm20x1dxo is the PWFO output files abbreviation, First OOS Day End Date (12/09/19), Last OOS Day End Date (07/09/21), **Number of days** (#355) a=average of bootstrap random picks. s= standard deviation of bootstrap random picks. f=number of different filters examined. c= slippage and round-trip trade cost(c=\$4).

The WFINP AVE File Output Cols are defined as follows.

- **Row 2 to Last Row Columns: A through Z**

Col A: *The Strategy Input/Filter Names*

Row 3: 3|13|2|2|0|1555|23.6|pf<2|: The inputs **3|13|2|2|0|1555|23.6** for all in-sample files that have PF≤2.

Col B: *toGP* Total out-of-sample(oos) gross profit for these 355 oos periods (for this run periods = weeks).

Col C: *toNP* Total out-of-sample(oos) Net profit (toGP-Number of Trade Weeks*cost) for the 355 oos periods.

Col D: *aoGP* Average oos gross profit for the # oos periods

Col E: *aoTr* Average oos profit per trade

Col F: *ao#T* Average number of oos trades per week

Col G: *#* The number of oos periods this filter produced any profit or loss. Note for some oos periods there are no trades.

Col H: *std* the standard deviation of the # oos period profits and losses

Col I: *skew* The Skew statistic of the # oos period profits and losses.

Col J: *kur* the kurtosis statistic of the # oos period profits and losses

Col K: *t* the student t statistic for the # oos periods. The higher the t statistic the higher the probability that this result was not due to pure chance.

Col L: *oW|oL* Ratio of average oos winning trades divided by average oos losing trades.

Col M: *%Wtr* The percentage if oos winning trades

Col N: *%P* percent of all oos periods that were profitable.

Col O: *LLtr* the largest losing oos trade in all oos periods

Col P: *LLp* the largest losing oos period

Col Q: *eqDD* the oos equity drawdown

Col R: *wpr* the largest number of winning oos periods (weeks) in a row.

Col S: *lpr* the largest number of losing oos periods in a row.

can be no strategy inputs that satisfy a given filters criteria, and no trades will be made during that period.

Col T: *v20* the straight-line trend of the oos equity curve for the last 20 bars.

Col U: *KTau* The Kendall rank coefficient is often used as a test statistic in a statistical hypothesis test to establish whether two variables may be regarded as statistically dependent. This test is non-parametric, as it does not rely on any assumptions on the distributions of X or Y or the distribution of (X,Y)

Col V: *eqR2* the correlation coefficient(R²) of a straight line fit to the equity curve.

Col W: *Blw* The maximum number of oos periods the oos equity curve failed to make a new high.

Col X: *BE* Break even in oos periods. Assuming the average and standard deviation are from a normal distribution, this is the number of oos periods you would have to trade to have a 98% probability that your oos equity is above zero.

Col Y: *tkr/bl* $=100 \cdot t \cdot K \tau \cdot \text{eqR2} / \text{Blw} / \text{BE}$. This is measure of the best equity curve.

Col Z: *Prob* The probability that the filters oos toNP was due to pure chance. Row 1 lists the random bootstrap average for the 355 out-of-sample files of (\$6.4) with a bootstrap standard deviation of \$10.0. (Note. The average for the random selection is computed as the Average Random toNP/355) The average net weekly for the filter would be the filter toNP/ (# of OOS) periods traded would be the filter toNP/# of OOS periods traded or $10726/182=58.93$). The probability of obtaining our filters average daily net profit of **58.93** is 2.68^{-11} which is 6.5 standard deviations from the bootstrap average. For our filter, in row 3 above, the expected number of cases that we could obtain by pure chance that would match or exceed **\$58.93** is $[1 - (2.28 \times 10^{-11})^{599759}] \sim 599759 \cdot 2.68 \times 10^{-11} = 0.0000161$ where **599759** is the total number of different filters we looked at in this run. This number is much much less than one, so it is improbable that our result was due to pure chance.

Table 1 Walk Forward Out-Of-Sample Performance Summary for the IWM 5-min FixmVn Strategy

IWM-5 min bars 02/01/2022 - 6/30/2023.

Filter: 3|12|2|2|0|1555|23.6|pf<2|; The inputs 3|13|2|2|0|1555|23.6 for all in-sample files that have PF≤2. are used to trade in the following out-of-sample sections.

IS-pf = In-sample pf

osnp = Daily out-of-sample gross profit in \$

NOnp\$4 = Daily out-of-sample net profit in \$ = osnp-ont*4.

ont = The number of trades in the out-of-sample day

ownp = winning profits in the out-of-sample day.

ownt = number of winning trades in the out-of-sample day

ollt = The largest losing trade in the out-of-sample day in \$.

odd = The drawdown in the out-of-sample day in \$.

EQ=Equity = Running Sum of daily out-of-sample gross profits \$

NetEq=Net Equity = running sum of the daily out-of-sample net profits in \$

Note: Blank rows indicate that no out-of-sample trades were made that day

OSS Day Date	IS pf	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
2/1/2022	1.91	440	432	2	440	2	0	0	440	432
2/2/2022	2.09	0	0	0	0	0	0	0	440	432
2/3/2022	1.75	-592	-600	2	0	0	-374	-592	-152	-168
2/4/2022	1.55	-262	-266	1	0	0	-262	-262	-414	-434
2/7/2022	1.43	-93	-101	2	0	0	-90	-93	-507	-535
2/8/2022	1.35	0	0	0	0	0	0	0	-507	-535
2/9/2022	1.34	-42	-50	2	34	1	-76	-76	-549	-585
2/10/2022	1.28	295	283	3	520	2	-225	-225	-254	-302
2/11/2022	1.25	-91	-103	3	77	1	-144	-144	-345	-405
2/14/2022	1.25	24	16	2	48	1	-24	-24	-321	-389
2/15/2022	1.26	225	213	3	253	2	-28	-28	-96	-176
2/16/2022	1.36	-209	-213	1	0	0	-209	-209	-305	-389
2/17/2022	1.5	326	322	1	326	1	0	0	21	-67
2/18/2022	1.86	98	90	2	208	1	-110	-110	119	23
2/22/2022	1.57	-153	-165	3	205	1	-194	-358	-34	-142
2/23/2022	1.34	318	310	2	443	1	-125	-125	284	168
2/24/2022	1.22	-183	-215	8	468	2	-185	-599	101	-47
2/25/2022	1.09	253	245	2	389	1	-136	-136	354	198
2/28/2022	0.96	24	16	2	123	1	-99	-99	378	214
3/1/2022	0.94	369	365	1	369	1	0	0	747	579
3/2/2022	0.92	354	342	3	354	3	0	0	1101	921
3/3/2022	1.21	0	0	0	0	0	0	0	1101	921
3/4/2022	1.49	-76	-84	2	23	1	-99	-99	1025	837
3/7/2022	1.6	305	301	1	305	1	0	0	1330	1138
3/8/2022	1.8	599	583	4	599	4	0	0	1929	1721
3/9/2022	2.06	0	0	0	0	0	0	0	1929	1721
3/10/2022	2.11	0	0	0	0	0	0	0	1929	1721
3/11/2022	1.75	350	342	2	400	1	-50	-50	2279	2063
3/14/2022	1.98	340	336	1	340	1	0	0	2619	2399

OSS Day Date	IS pf	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
3/15/2022	2.12	0	0	0	0	0	0	0	2619	2399
3/16/2022	1.87	267	247	5	485	3	-134	-218	2886	2646
3/17/2022	2.05	0	0	0	0	0	0	0	2886	2646
3/18/2022	1.93	136	132	1	136	1	0	0	3022	2778
3/21/2022	1.98	-285	-293	2	0	0	-195	-285	2737	2485
3/22/2022	1.78	88	84	1	88	1	0	0	2825	2569
3/23/2022	1.99	-39	-47	2	72	1	-111	-111	2786	2522
3/24/2022	1.85	0	0	0	0	0	0	0	2786	2522
3/25/2022	2.28	0	0	0	0	0	0	0	2786	2522
3/28/2022	2.23	0	0	0	0	0	0	0	2786	2522
3/29/2022	2.19	0	0	0	0	0	0	0	2786	2522
3/30/2022	2.07	0	0	0	0	0	0	0	2786	2522
3/31/2022	1.84	0	0	0	0	0	0	0	2786	2522
4/1/2022	1.84	6	2	1	6	1	0	0	2792	2524
4/4/2022	1.96	0	0	0	0	0	0	0	2792	2524
4/5/2022	1.75	447	443	1	447	1	0	0	3239	2967
4/6/2022	1.65	10	2	2	40	1	-30	-30	3249	2969
4/7/2022	1.63	0	0	0	0	0	0	0	3249	2969
4/8/2022	2.23	0	0	0	0	0	0	0	3249	2969
4/11/2022	1.95	228	220	2	228	2	0	0	3477	3189
4/12/2022	1.84	584	576	2	584	2	0	0	4061	3765
4/13/2022	3.1	0	0	0	0	0	0	0	4061	3765
4/14/2022	2.21	0	0	0	0	0	0	0	4061	3765
4/18/2022	2.05	0	0	0	0	0	0	0	4061	3765
4/19/2022	3.08	0	0	0	0	0	0	0	4061	3765
4/20/2022	3.54	0	0	0	0	0	0	0	4061	3765
4/21/2022	3.57	0	0	0	0	0	0	0	4061	3765
4/22/2022	1.58	323	319	1	323	1	0	0	4384	4084
4/25/2022	1.85	4	-12	4	290	1	-128	-286	4388	4072
4/26/2022	1.78	-386	-394	2	22	1	-408	-408	4002	3678
4/27/2022	1.28	17	9	2	30	1	-13	-13	4019	3687
4/28/2022	1.28	-327	-331	1	0	0	-327	-327	3692	3356
4/29/2022	1.09	505	501	1	505	1	0	0	4197	3857
5/2/2022	1.32	9	-7	4	311	2	-223	-302	4206	3850
5/3/2022	1.28	63	59	1	63	1	0	0	4269	3909
5/4/2022	1.13	79	63	4	362	1	-144	-149	4348	3972
5/5/2022	1.14	-119	-135	4	219	2	-244	-244	4229	3837
5/6/2022	1.09	-35	-59	6	212	3	-185	-247	4194	3778
5/9/2022	1.07	541	537	1	541	1	0	0	4735	4315
5/10/2022	1.16	496	476	5	739	2	-137	-164	5231	4791
5/11/2022	1.13	398	382	4	535	1	-69	-137	5629	5173
5/12/2022	1.34	273	257	4	363	2	-51	-90	5902	5430
5/13/2022	1.42	62	50	3	156	2	-94	-94	5964	5480
5/16/2022	1.42	141	133	2	141	2	0	0	6105	5613
5/17/2022	1.46	83	71	3	209	1	-97	-126	6188	5684
5/18/2022	1.37	322	310	3	447	1	-75	-125	6510	5994
5/19/2022	1.48	-645	-661	4	0	0	-263	-645	5865	5333
5/20/2022	1.49	397	389	2	397	2	0	0	6262	5722
5/23/2022	1.51	55	47	2	112	1	-57	-57	6317	5769

OSS Day Date	IS pf	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
5/24/2022	1.56	173	165	2	173	2	0	0	6490	5934
5/25/2022	1.82	0	0	0	0	0	0	0	6490	5934
5/26/2022	1.82	171	167	1	171	1	0	0	6661	6101
5/27/2022	2.1	0	0	0	0	0	0	0	6661	6101
5/31/2022	2.03	0	0	0	0	0	0	0	6661	6101
6/1/2022	2.01	0	0	0	0	0	0	0	6661	6101
6/2/2022	2.11	0	0	0	0	0	0	0	6661	6101
6/3/2022	2.39	0	0	0	0	0	0	0	6661	6101
6/6/2022	2.47	0	0	0	0	0	0	0	6661	6101
6/7/2022	2.14	0	0	0	0	0	0	0	6661	6101
6/8/2022	1.98	152	148	1	152	1	0	0	6813	6249
6/9/2022	1.89	131	119	3	179	2	-48	-48	6944	6368
6/10/2022	1.8	155	151	1	155	1	0	0	7099	6519
6/13/2022	1.95	304	284	5	424	3	-112	-120	7403	6803
6/14/2022	1.99	-268	-284	4	0	0	-112	-268	7135	6519
6/15/2022	1.63	-338	-354	4	60	1	-156	-398	6797	6165
6/16/2022	1.13	-39	-47	2	86	1	-125	-125	6758	6118
6/17/2022	1.72	222	214	2	222	2	0	0	6980	6332
6/21/2022	1.55	111	107	1	111	1	0	0	7091	6439
6/22/2022	1.49	-173	-177	1	0	0	-173	-173	6918	6262
6/23/2022	1.28	-228	-232	1	0	0	-228	-228	6690	6030
6/24/2022	0.97	308	304	1	308	1	0	0	6998	6334
6/27/2022	1.18	-68	-72	1	0	0	-68	-68	6930	6262
6/28/2022	1.13	419	411	2	437	1	-18	-18	7349	6673
6/29/2022	1.39	-99	-115	4	8	1	-57	-107	7250	6558
6/30/2022	1.3	190	182	2	190	2	0	0	7440	6740
7/1/2022	1.42	-158	-166	2	0	0	-103	-158	7282	6574
7/5/2022	1.36	643	635	2	643	2	0	0	7925	7209
7/6/2022	1.74	77	73	1	77	1	0	0	8002	7282
7/7/2022	1.69	88	76	3	118	2	-30	-30	8090	7358
7/8/2022	1.68	58	54	1	58	1	0	0	8148	7412
7/11/2022	1.62	-68	-76	2	58	1	-126	-126	8080	7336
7/12/2022	1.4	-70	-74	1	0	0	-70	-70	8010	7262
7/13/2022	1.58	-91	-99	2	49	1	-140	-140	7919	7163
7/14/2022	1.9	-77	-81	1	0	0	-77	-77	7842	7082
7/15/2022	1.91	-89	-101	3	232	1	-218	-321	7753	6981
7/18/2022	1.51	-290	-302	3	27	1	-301	-301	7463	6679
7/19/2022	1.26	181	169	3	252	2	-71	-71	7644	6848
7/20/2022	1.29	0	0	0	0	0	0	0	7644	6848
7/21/2022	1.42	46	38	2	169	1	-123	-123	7690	6886
7/22/2022	1.62	-161	-169	2	20	1	-181	-181	7529	6717
7/25/2022	1.29	0	0	0	0	0	0	0	7529	6717
7/26/2022	1.34	0	0	0	0	0	0	0	7529	6717
7/27/2022	1.1	105	101	1	105	1	0	0	7634	6818
7/28/2022	1.24	-315	-319	1	0	0	-315	-315	7319	6499
7/29/2022	0.94	0	0	0	0	0	0	0	7319	6499
8/1/2022	1.02	-240	-260	5	5	1	-123	-240	7079	6239
8/2/2022	0.9	0	0	0	0	0	0	0	7079	6239
8/3/2022	0.58	0	0	0	0	0	0	0	7079	6239

OSS Day Date	IS pf	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
8/4/2022	0.54	0	0	0	0	0	0	0	7079	6239
8/5/2022	0.49	179	175	1	179	1	0	0	7258	6414
8/8/2022	0.55	-27	-31	1	0	0	-27	-27	7231	6383
8/9/2022	0.55	177	173	1	177	1	0	0	7408	6556
8/10/2022	0.67	-134	-142	2	44	1	-178	-178	7274	6414
8/11/2022	0.65	-100	-104	1	0	0	-100	-100	7174	6310
8/12/2022	0.64	0	0	0	0	0	0	0	7174	6310
8/15/2022	0.63	0	0	0	0	0	0	0	7174	6310
8/16/2022	0.77	0	0	0	0	0	0	0	7174	6310
8/17/2022	0.6	14	6	2	48	1	-34	-34	7188	6316
8/18/2022	0.62	0	0	0	0	0	0	0	7188	6316
8/19/2022	0.54	148	140	2	179	1	-31	-31	7336	6456
8/22/2022	0.79	-117	-129	3	39	1	-109	-156	7219	6327
8/23/2022	0.71	0	0	0	0	0	0	0	7219	6327
8/24/2022	0.71	0	0	0	0	0	0	0	7219	6327
8/25/2022	0.62	0	0	0	0	0	0	0	7219	6327
8/26/2022	0.87	362	358	1	362	1	0	0	7581	6685
8/29/2022	1.34	0	0	0	0	0	0	0	7581	6685
8/30/2022	1.95	0	0	0	0	0	0	0	7581	6685
8/31/2022	1.95	0	0	0	0	0	0	0	7581	6685
9/1/2022	1.95	226	218	2	226	2	0	0	7807	6903
9/2/2022	2.38	0	0	0	0	0	0	0	7807	6903
9/6/2022	1.43	48	44	1	48	1	0	0	7855	6947
9/7/2022	1.26	0	0	0	0	0	0	0	7855	6947
9/8/2022	1.57	26	18	2	193	1	-167	-167	7881	6965
9/9/2022	1.71	161	157	1	161	1	0	0	8042	7122
9/12/2022	1.96	0	0	0	0	0	0	0	8042	7122
9/13/2022	1.96	270	258	3	299	2	-29	-29	8312	7380
9/14/2022	2.32	0	0	0	0	0	0	0	8312	7380
9/15/2022	2.39	0	0	0	0	0	0	0	8312	7380
9/16/2022	2.63	0	0	0	0	0	0	0	8312	7380
9/19/2022	2.83	0	0	0	0	0	0	0	8312	7380
9/20/2022	3.73	0	0	0	0	0	0	0	8312	7380
9/21/2022	3.78	0	0	0	0	0	0	0	8312	7380
9/22/2022	5.34	0	0	0	0	0	0	0	8312	7380
9/23/2022	4.07	0	0	0	0	0	0	0	8312	7380
9/26/2022	3.78	0	0	0	0	0	0	0	8312	7380
9/27/2022	4.27	0	0	0	0	0	0	0	8312	7380
9/28/2022	2.71	0	0	0	0	0	0	0	8312	7380
9/29/2022	2.71	0	0	0	0	0	0	0	8312	7380
9/30/2022	2.7	0	0	0	0	0	0	0	8312	7380
10/3/2022	2.86	0	0	0	0	0	0	0	8312	7380
10/4/2022	3.09	0	0	0	0	0	0	0	8312	7380
10/5/2022	2.9	0	0	0	0	0	0	0	8312	7380
10/6/2022	2.57	0	0	0	0	0	0	0	8312	7380
10/7/2022	2.88	0	0	0	0	0	0	0	8312	7380
10/10/2022	2.62	0	0	0	0	0	0	0	8312	7380
10/11/2022	2.48	0	0	0	0	0	0	0	8312	7380
10/12/2022	2.12	0	0	0	0	0	0	0	8312	7380

OSS Day Date	IS pf	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
10/13/2022	1.84	366	350	4	582	1	-161	-216	8678	7730
10/14/2022	1.86	218	210	2	435	1	-217	-217	8896	7940
10/17/2022	1.75	48	36	3	101	2	-53	-53	8944	7976
10/18/2022	1.75	76	68	2	155	1	-79	-79	9020	8044
10/19/2022	1.75	-128	-136	2	0	0	-122	-128	8892	7908
10/20/2022	1.28	361	353	2	361	2	0	0	9253	8261
10/21/2022	1.58	344	336	2	344	2	0	0	9597	8597
10/24/2022	1.66	0	0	0	0	0	0	0	9597	8597
10/25/2022	1.5	325	321	1	325	1	0	0	9922	8918
10/26/2022	2.06	0	0	0	0	0	0	0	9922	8918
10/27/2022	2.06	0	0	0	0	0	0	0	9922	8918
10/28/2022	1.63	256	244	3	366	2	-110	-110	10178	9162
10/31/2022	1.92	0	0	0	0	0	0	0	10178	9162
11/1/2022	1.8	-139	-147	2	0	0	-85	-139	10039	9015
11/2/2022	1.69	-180	-200	5	155	3	-331	-331	9859	8815
11/3/2022	1.59	122	114	2	154	1	-32	-32	9981	8929
11/4/2022	1.62	-18	-22	1	0	0	-18	-18	9963	8907
11/7/2022	1.59	-159	-163	1	0	0	-159	-159	9804	8744
11/8/2022	1.51	0	0	0	0	0	0	0	9804	8744
11/9/2022	1.56	367	355	3	376	2	-9	-9	10171	9099
11/10/2022	1.89	300	288	3	352	2	-52	-52	10471	9387
11/11/2022	1.93	0	0	0	0	0	0	0	10471	9387
11/14/2022	1.92	-125	-133	2	0	0	-74	-125	10346	9254
11/15/2022	1.77	-101	-113	3	23	1	-97	-124	10245	9141
11/16/2022	1.64	135	131	1	135	1	0	0	10380	9272
11/17/2022	1.87	136	128	2	136	2	0	0	10516	9400
11/18/2022	1.72	0	0	0	0	0	0	0	10516	9400
11/21/2022	1.49	0	0	0	0	0	0	0	10516	9400
11/22/2022	1.49	0	0	0	0	0	0	0	10516	9400
11/23/2022	1.27	0	0	0	0	0	0	0	10516	9400
11/25/2022	1.54	0	0	0	0	0	0	0	10516	9400
11/28/2022	1.34	0	0	0	0	0	0	0	10516	9400
11/29/2022	1.34	0	0	0	0	0	0	0	10516	9400
11/30/2022	1.56	102	94	2	232	1	-130	-130	10618	9494
12/1/2022	2.17	0	0	0	0	0	0	0	10618	9494
12/2/2022	1.68	104	96	2	199	1	-95	-95	10722	9590
12/5/2022	1.76	302	298	1	302	1	0	0	11024	9888
12/6/2022	2.64	0	0	0	0	0	0	0	11024	9888
12/7/2022	2.64	0	0	0	0	0	0	0	11024	9888
12/8/2022	2.1	0	0	0	0	0	0	0	11024	9888
12/9/2022	1.7	0	0	0	0	0	0	0	11024	9888
12/12/2022	1.7	0	0	0	0	0	0	0	11024	9888
12/13/2022	2.14	0	0	0	0	0	0	0	11024	9888
12/14/2022	4.12	0	0	0	0	0	0	0	11024	9888
12/15/2022	2.39	0	0	0	0	0	0	0	11024	9888
12/16/2022	2.51	0	0	0	0	0	0	0	11024	9888
12/19/2022	2.51	0	0	0	0	0	0	0	11024	9888
12/20/2022	2.51	0	0	0	0	0	0	0	11024	9888
12/21/2022	2.51	0	0	0	0	0	0	0	11024	9888

OSS Day Date	IS pf	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
12/22/2022	2.51	0	0	0	0	0	0	0	11024	9888
12/23/2022	2.36	0	0	0	0	0	0	0	11024	9888
12/27/2022	2.45	0	0	0	0	0	0	0	11024	9888
12/28/2022	2.45	0	0	0	0	0	0	0	11024	9888
12/29/2022	2.64	0	0	0	0	0	0	0	11024	9888
12/30/2022	4.49	0	0	0	0	0	0	0	11024	9888
1/3/2023	2.32	0	0	0	0	0	0	0	11024	9888
1/4/2023	1.75	-132	-140	2	0	0	-81	-132	10892	9748
1/5/2023	1.45	45	33	3	92	2	-47	-47	10937	9781
1/6/2023	1.48	171	167	1	171	1	0	0	11108	9948
1/9/2023	1.69	0	0	0	0	0	0	0	11108	9948
1/10/2023	1.69	153	149	1	153	1	0	0	11261	10097
1/11/2023	1.3	0	0	0	0	0	0	0	11261	10097
1/12/2023	1.74	234	230	1	234	1	0	0	11495	10327
1/13/2023	1.79	0	0	0	0	0	0	0	11495	10327
1/17/2023	1.79	0	0	0	0	0	0	0	11495	10327
1/18/2023	1.79	0	0	0	0	0	0	0	11495	10327
1/19/2023	1.79	0	0	0	0	0	0	0	11495	10327
1/20/2023	1.9	0	0	0	0	0	0	0	11495	10327
1/23/2023	1.81	0	0	0	0	0	0	0	11495	10327
1/24/2023	1.81	0	0	0	0	0	0	0	11495	10327
1/25/2023	1.81	-241	-245	1	0	0	-241	-241	11254	10082
1/26/2023	1.27	-218	-230	3	48	1	-203	-266	11036	9852
1/27/2023	0.76	0	0	0	0	0	0	0	11036	9852
1/30/2023	0.91	6	-6	3	163	1	-118	-157	11042	9846
1/31/2023	0.93	91	83	2	218	1	-127	-127	11133	9929
2/1/2023	1.11	59	55	1	59	1	0	0	11192	9984
2/2/2023	1.36	-7	-19	3	127	1	-70	-134	11185	9965
2/3/2023	1.27	0	0	0	0	0	0	0	11185	9965
2/6/2023	1.08	-41	-45	1	0	0	-41	-41	11144	9920
2/7/2023	1.04	525	513	3	560	2	-35	-35	11669	10433
2/8/2023	1.41	0	0	0	0	0	0	0	11669	10433
2/9/2023	1.41	346	342	1	346	1	0	0	12015	10775
2/10/2023	1.52	0	0	0	0	0	0	0	12015	10775
2/13/2023	1.52	0	0	0	0	0	0	0	12015	10775
2/14/2023	1.52	-472	-492	5	93	1	-241	-472	11543	10283
2/15/2023	1.03	197	189	2	261	1	-64	-64	11740	10472
2/16/2023	1.15	-105	-109	1	0	0	-105	-105	11635	10363
2/17/2023	1.08	108	104	1	108	1	0	0	11743	10467
2/21/2023	1.14	370	366	1	370	1	0	0	12113	10833
2/22/2023	1.36	35	31	1	35	1	0	0	12148	10864
2/23/2023	1.6	-50	-58	2	3	1	-53	-53	12098	10806
2/24/2023	1.83	-276	-288	3	0	0	-133	-276	11822	10518
2/27/2023	1.5	-63	-67	1	0	0	-63	-63	11759	10451
2/28/2023	1.49	0	0	0	0	0	0	0	11759	10451
3/1/2023	1.47	-58	-62	1	0	0	-58	-58	11701	10389
3/2/2023	1.37	-239	-251	3	0	0	-201	-239	11462	10138
3/3/2023	1.18	0	0	0	0	0	0	0	11462	10138
3/6/2023	1.18	0	0	0	0	0	0	0	11462	10138

OSS Day Date	IS pf	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
3/7/2023	1.22	56	52	1	56	1	0	0	11518	10190
3/8/2023	0.89	0	0	0	0	0	0	0	11518	10190
3/9/2023	0.89	0	0	0	0	0	0	0	11518	10190
3/10/2023	0.65	143	115	7	380	2	-88	-137	11661	10305
3/13/2023	0.79	-71	-91	5	260	2	-134	-331	11590	10214
3/14/2023	0.79	107	91	4	212	1	-50	-105	11697	10305
3/15/2023	1.1	21	13	2	31	1	-10	-10	11718	10318
3/16/2023	0.99	375	371	1	375	1	0	0	12093	10689
3/17/2023	1.33	-56	-68	3	85	1	-127	-141	12037	10621
3/20/2023	1.19	57	53	1	57	1	0	0	12094	10674
3/21/2023	1.23	61	53	2	61	2	0	0	12155	10727
3/22/2023	1.03	-370	-390	5	131	1	-200	-501	11785	10337
3/23/2023	0.82	-155	-159	1	0	0	-155	-155	11630	10178
3/24/2023	0.78	201	193	2	246	1	-45	-45	11831	10371
3/27/2023	1	56	52	1	56	1	0	0	11887	10423
3/28/2023	1.07	0	0	0	0	0	0	0	11887	10423
3/29/2023	1.07	0	0	0	0	0	0	0	11887	10423
3/30/2023	1.11	78	74	1	78	1	0	0	11965	10497
3/31/2023	1.33	103	99	1	103	1	0	0	12068	10596
4/3/2023	1.4	-47	-51	1	0	0	-47	-47	12021	10545
4/4/2023	1.36	0	0	0	0	0	0	0	12021	10545
4/5/2023	1.32	0	0	0	0	0	0	0	12021	10545
4/6/2023	1.32	0	0	0	0	0	0	0	12021	10545
4/10/2023	1.27	0	0	0	0	0	0	0	12021	10545
4/11/2023	1.43	0	0	0	0	0	0	0	12021	10545
4/12/2023	1.36	-271	-275	1	0	0	-271	-271	11750	10270
4/13/2023	1.03	0	0	0	0	0	0	0	11750	10270
4/14/2023	0.7	0	0	0	0	0	0	0	11750	10270
4/17/2023	0.72	0	0	0	0	0	0	0	11750	10270
4/18/2023	0.66	0	0	0	0	0	0	0	11750	10270
4/19/2023	0.6	0	0	0	0	0	0	0	11750	10270
4/20/2023	0.93	0	0	0	0	0	0	0	11750	10270
4/21/2023	1.33	0	0	0	0	0	0	0	11750	10270
4/24/2023	0.75	0	0	0	0	0	0	0	11750	10270
4/25/2023	0.57	239	235	1	239	1	0	0	11989	10505
4/26/2023	1.32	0	0	0	0	0	0	0	11989	10505
4/27/2023	1.32	0	0	0	0	0	0	0	11989	10505
4/28/2023	1.08	0	0	0	0	0	0	0	11989	10505
5/1/2023	0.75	0	0	0	0	0	0	0	11989	10505
5/2/2023	0.88	183	179	1	183	1	0	0	12172	10684
5/3/2023	1.56	0	0	0	0	0	0	0	12172	10684
5/4/2023	1.56	-200	-216	4	22	2	-115	-222	11972	10468
5/5/2023	0.9	65	53	3	91	1	-21	-26	12037	10521
5/8/2023	1.03	0	0	0	0	0	0	0	12037	10521
5/9/2023	1.03	-94	-98	1	0	0	-94	-94	11943	10423
5/10/2023	0.87	-105	-109	1	0	0	-105	-105	11838	10314
5/11/2023	1.2	7	3	1	7	1	0	0	11845	10317
5/12/2023	1.21	38	34	1	38	1	0	0	11883	10351
5/15/2023	1.3	0	0	0	0	0	0	0	11883	10351

OSS Day Date	IS pf	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
5/16/2023	1.3	61	57	1	61	1	0	0	11944	10408
5/17/2023	1.43	0	0	0	0	0	0	0	11944	10408
5/18/2023	1.43	0	0	0	0	0	0	0	11944	10408
5/19/2023	1.43	0	0	0	0	0	0	0	11944	10408
5/22/2023	1.43	0	0	0	0	0	0	0	11944	10408
5/23/2023	1.43	0	0	0	0	0	0	0	11944	10408
5/24/2023	0.9	0	0	0	0	0	0	0	11944	10408
5/25/2023	0.9	0	0	0	0	0	0	0	11944	10408
5/26/2023	0.9	0	0	0	0	0	0	0	11944	10408
5/30/2023	0.9	0	0	0	0	0	0	0	11944	10408
5/31/2023	0.49	0	0	0	0	0	0	0	11944	10408
6/1/2023	0.49	0	0	0	0	0	0	0	11944	10408
6/2/2023	0.88	243	231	3	351	1	-70	-108	12187	10639
6/5/2023	1.49	0	0	0	0	0	0	0	12187	10639
6/6/2023	1.49	0	0	0	0	0	0	0	12187	10639
6/7/2023	2.15	0	0	0	0	0	0	0	12187	10639
6/8/2023	5.08	0	0	0	0	0	0	0	12187	10639
6/9/2023	5.02	0	0	0	0	0	0	0	12187	10639
6/12/2023	4.67	0	0	0	0	0	0	0	12187	10639
6/13/2023	4.67	0	0	0	0	0	0	0	12187	10639
6/14/2023	4.75	0	0	0	0	0	0	0	12187	10639
6/15/2023	3.4	0	0	0	0	0	0	0	12187	10639
6/16/2023	3.4	0	0	0	0	0	0	0	12187	10639
6/20/2023	3.4	0	0	0	0	0	0	0	12187	10639
6/21/2023	3.4	0	0	0	0	0	0	0	12187	10639
6/22/2023	3.4	0	0	0	0	0	0	0	12187	10639
6/23/2023	3.48	0	0	0	0	0	0	0	12187	10639
6/26/2023	1.25	23	15	2	62	1	-39	-39	12210	10654
6/27/2023	1.28	0	0	0	0	0	0	0	12210	10654
6/28/2023	1.28	0	0	0	0	0	0	0	12210	10654
6/29/2023	1.28	76	72	1	76	1	0	0	12286	10726
6/30/2023	1.44	0	0	0	0	0	0	0	12286	10726

Appendix I: n^{th} Order Polynomial Next Bar's Forecast Math

What is the n^{th} Order Polynomial?

The n^{th} Order Polynomial, also called the n^{th} Order Fixed Memory Polynomial, is simply the least square fit of a polynomial of the form $b_0 + b_1 * t + b_2 * t^2 + b_3 * t^3 + \dots + b_n * t^n$ to a *fixed* number of past data points. Where t is discrete time bars. Time could be daily bars or 5-minute bars. We use the term "Fixed Memory" to designate that only a fixed number of data points are used to calculate the polynomial coefficients. It is assumed that the time bars occur at fixed intervals of time so tic bars would not be appropriate for this analysis. Least squares are a mathematical technique where the squared vertical distance between the data and the curve that is being fit to the data is minimized. When the net squared distance (also called the sum of the squared errors) is minimized, a unique set of coefficients $b_0, b_1, b_2, \dots, b_n$ for the polynomial is determined. This type of error minimization is mathematically solvable and is widely used in science and mathematics.

For a 4th order polynomial equation, the least squares coefficients are obtained from the solution of the following matrix equation.

$$\begin{bmatrix} T & \sum t & \sum t^2 & \sum t^3 & \sum t^4 \\ \sum t & \sum t^2 & \sum t^3 & \sum t^4 & \sum t^5 \\ \sum t^2 & \sum t^3 & \sum t^4 & \sum t^5 & \sum t^6 \\ \sum t^3 & \sum t^4 & \sum t^5 & \sum t^6 & \sum t^7 \\ \sum t^4 & \sum t^5 & \sum t^6 & \sum t^7 & \sum t^8 \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \\ e_0 \end{bmatrix} = \begin{bmatrix} \sum p(t) \\ \sum (p(t) * t) \\ \sum (p(t) * t^2) \\ \sum (p(t) * t^3) \\ \sum (p(t) * t^4) \end{bmatrix}$$

where

$p(T)$ is the current bar's price, $p(T-1)$ is the previous bar's price and $p(1)$ is the price T bars ago.

T is the number of Bars in the Least Squares estimation

$\sum p(t)$ is the summation of prices from $t=1$ to T bars

$\sum p(t) * t$ is the summation of prices times t from $t=1$ to T bars

$\sum t$ is the summation of the integer t from $t=1$ to T bars

$\sum t^2$ is the summation of the integer t squared from $t=1$ to T bars

etc.

Once the coefficients to the polynomial have been solved for, we generate the forecast for the next bar's price which is given for the equation by:

$$P_f = a_0 + b_0 * (T+1) + c_0 * (T+1)^2 + d_0 * (T+1)^3 + e_0 * (T+1)^4$$

Where P_f stands for price forecast.

With these coefficients, we can also generate the forecast for the next bar's *velocity* and *velocity* by the equations:

$$\text{Velocity}(T+1) = dP_f / dt = b_0 + 2c_0 * (T+1) + 3d_0 * (T+1)^2 + 4e_0 * (T+1)^3$$

$$\text{Velocity}(t+1) = d^2P_f / d^2t = 2c_0 + 6d_0 * (T+1) + 12e_0 * (T+1)^2$$

We use the next bar forecast because changes in the trend are more quickly reflected in the forecast price, velocity and velocity than in the end point price, velocity and velocity.

Programs that solve for the solution to matrix equations can be found in the book "Numerical Recipes" by W. Press, et. al. However this type of matrix equation solvers is very slow and for these types of problems are unstable. They cause numerical errors and floating-point overflows due to matrix inversion ill conditioning which produces false results.

Appendix I: n^{th} Order Polynomial Next Bar's Forecast Math

Fortunately, these types of problems can be solved by a fast, efficient and accurate algorithm using Discrete Orthogonal Legendre Polynomials. This method is explained in detail in Norman Morrison's book entitled "Introduction to Sequential Smoothing and Prediction", Chapter 7 page 223., referenced at the end of this section.

Without going into detail here (see Morrison reference), the polynomial filter can now be represented by:

$$P_e(t) = \sum_{j=0}^n \beta_j \phi_j(t) \quad j=0 \text{ to } n$$

Where n is the polynomial order, T is the total number of Bars of data used in the Least Squares fit and

$$\beta_j = \sum_{k=0}^{T-1} p(t-T+k) \phi_j(k)$$

$\phi_j(t)$ = the *normalized discrete Legendre polynomial*. t = an integer from 0 to T

The coefficients, $\beta_0, \beta_1, \beta_2, \beta_3, \dots, \beta_n$ for a n^{th} order polynomial can now be solved for by the equation above and we can generate the forecast for the next bar's close, velocity and velocity which are given by the equations

$$P_f(T+1) = \beta_0 \phi_0(T+1) + \beta_1 \phi_1(T+1) + \beta_2 \phi_2(T+1) + \beta_3 \phi_3(T+1) + \dots + \beta_n \phi_n(T+1)$$

$$\text{Velocity} = (dP_f/dt)_{(T+1)} = \beta_1 (d\phi_1/dt)_{(T+1)} + \beta_2 (d\phi_2/dt)_{(T+1)} + \beta_3 (d\phi_3/dt)_{(T+1)} + \dots + \beta_n (d\phi_n/dt)_{(T+1)}$$

$$\text{Velocity} = (d^2P_f/d^2t)_{(T+1)} = \beta_2 (d^2\phi_2/d^2t)_{(T+1)} + \beta_3 (d^2\phi_3/d^2t)_{(T+1)} + \dots + \beta_n (d^2\phi_n/d^2t)_{(T+1)}$$

The n^{th} Order Fixed Memory Forecast Next Bar's Velocity Strategy Defined

The least squares forecast is constructed by solving for the least squares coefficients $\beta_1, \beta_2, \dots, \beta_n$ at each bar using the last T bars of closing prices and the Discrete Orthogonal Legendre Polynomial equations for β_j above. Then **Velocity** = $d^2P_f(T+1)/d^2t$ is constructed from the velocity equation above and plotted under the price chart. In general, what we will be doing is following the plotted curve of **Velocity** which is calculated at each bar from the previous T bars. When the velocity is greater than a threshold amount **vup** we will go long. When the velocity is less than a threshold amount **vdn** we will go short.

Buy Rule:

IF **Velocity** is greater than the threshold amount **vup** then buy at the market.

Sell Rule:

IF **Velocity** is less than the threshold amount **vdn** then sell at the market.

References

1. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.

The Normalization Multiplier

What is the Multiplier?

The n^{th} Order Fixed Memory Polynomial, also called an n^{th} Order Polynomial, is the least square fit of a polynomial of the form $b_0 + b_1 * t + b_2 * t^2 + b_3 * t^3 + \dots + b_n * t^n$ to a *fixed* number of past data points. Where t is discrete time bars. Time could be daily bars or 5-minute bars. We use the term “Fixed Memory” to designate that only a fixed number of data points are used to calculate the polynomial coefficients. It is assumed that the time bars occur at fixed intervals of time so tic bars would not be appropriate for this analysis. Least squares are a mathematical technique where the squared vertical distance between the data and the curve that is being fit to the data is minimized. When the net squared distance (also called the sum of the squared errors) is minimized, a unique set of coefficients $b_0, b_1, b_2, \dots, b_n$ for the polynomial is determined. This type of error minimization is mathematically solvable and is widely used in science and mathematics. Once the b_n coefficients are found using a lookback period of T bars to calculate the b_n coefficients, then the next bar’s estimate $(T+1)$ of the n^{th} order polynomial velocity and acceleration can be easily found by the equations below.

$$\text{Velocity}(T+1) = dP_f / dt = b_0 + 2c_0 * (T+1) + 3d_0 * (T+1)^2 + 4e_0 * (T+1)^3 + \dots + n * b_n * (T+1)^{n-1}$$

$$\text{Acceleration}(t+1) = d^2P_f / d^2t = 2 c_0 + 9d_0 * (T+1) + 12e_0 * (T+1)^3 + \dots + n * (n-1) * b_n * (T+1)^{n-2}$$

Please see the n^{th} Order Fixed Memory Polynomial Next Bar’s Forecast Math section for a more detailed explanation.

For any tradable, the inputs to the polynomial are the **polynomial degree (Order)** and the number or lookback bars N (denoted by T in equations above). When we plot the velocity or acceleration, we notice that the amplitude, and the maximum and minimum values of the velocity or acceleration vary quite significantly with different degree and N inputs.

Below is a table of the standard deviation (SD) of the 56340 calculated Velocity values for different **degree** and **N** inputs. We used 1min bars of the E-Mini from 8/1/2014 to 2/20/2015 to generate this table. As one can see the standard deviation of the velocity for each degree and N vary greatly. For instance, for degree=4, $N=20$ the SD is 6.8 times the SD for degree=1, $N=20$. This creates problems when trying to determine the correct ranges for vup/vdn and aup/adn during optimization.

@ES.D 5 min bars Date Range 1140801 to 1150220

Total Number of Bars=56340 Norm=0

FixmVn Multiplier= 1/SD to Scale Velocity pw and N Range to One SD

Degree	N	SD	1/SD
1	20	0.1902	5.2565
1	30	0.1540	6.4916
1	40	0.1328	7.5279
1	50	0.1183	8.4502
1	60	0.1077	9.3550
1	70	0.0996	10.0440
avg		0.1338	7.8430
2	20	0.4351	2.2982
2	30	0.3443	2.9046
2	40	0.2936	3.4060
2	50	0.1583	3.8275
2	60	0.2371	4.2180
2	70	0.2173	4.6010

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avg		0.2981	3.5425
3	20	0.7854	1.2732
3	30	0.5933	1.6855
3	40	0.4973	2.0111
3	50	0.4347	2.3005
3	60	0.3949	2.5324
3	70	0.3656	2.7352
avg		0.5119	2.0897
4	20	1.2924	0.7738
4	30	0.9279	1.0777
4	40	0.7582	1.3189
4	5	0.6542	1.5285
4	60	0.5804	1.7228
4	70	0.5314	1.8818
avg		0.7908	1.3839

The problem may get worse when we want to find good inputs for other tradables. Other tradables, because of their scales and tick size have much different Velocity ranges than the E-Mini for the same degree and N. Thus, the NS search ranges have to be different for each different tradable.

To solve this problem and to have a standard search space for each tradable, I created a **Mult** input for each FixmXVA Velocity and Acceleration strategy and indicator. If each tradable's Velocity is multiplied by a number such that the standard deviation of that tradable's Velocity is around one, then the search space for vup and vdn for each tradable would be 0 to 3.5 SDs and we wouldn't have to change the TS search space every time we wanted to examine a new stock or future. The complicated equations that I use to normalize the ranges to one standard deviation were derived using the software TableCurve 3D, automated surface and equation discovery.