

Trading 1Min Bar E-Mini NASDAQ 100 Futures Using The Nth Order Fixed Memory Polynomial Velocity Strategy

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In previous working papers at <http://www.meyersanalytics.com/papers.php> we showed how the application of a price curve generated by the Nth Order Fixed Memory Polynomial Velocity could be used to develop a strategy to buy and sell futures intraday. The reasoning behind this type of strategy was to only trade when the price trend velocity was above a certain threshold. Many times prices meander around without any notable trend and this is considered noise. During these times we do not wish to trade because of the cost of whipsaw losses that would occur from this type of price action. When a price trend finally starts, the velocity of that price trend moves above a minimum threshold noise value. Thus, the velocity strategy would only issue a trade when certain velocity thresholds above “noise” levels are crossed.

The velocity strategy that we will use here to trade the E-Mini NASDAQ 100 futures(NQ)contract is called the nth Order Adaptive Polynomial Velocity Strategy. The word “Adaptive” is used because the polynomial inputs change over time, adapting to the changing trading patterns of the NQ futures contract. The nth Order Adaptive Polynomial Velocity Strategy has several unknown inputs that we must determine before we can use this strategy to trade. These unknown inputs are the polynomial order or degree, the optimum number of lookback prices we need to determine the co-efficients of the polynomial and finally the velocity thresholds. Here we will use Walk Forward Optimization and out-of-sample testing to determine the “best” polynomial inputs as well as how these inputs change over time. We will use the nth Order Fixed Memory Polynomial Velocity Strategy to trade the NQ futures contract on an intraday basis using one minute bar price data from January 3, 2014 to January 6, 2017 .

The nth Order Fixed Memory Velocity Strategy Defined

The least squares forecast nth order fixed memory polynomial velocity is constructed by solving for the coefficients $\beta_0, \beta_1, \beta_2, \beta_3 \dots \beta_n$ for the discrete orthogonal Legendre polynomials each day using the last **N bars** of closing prices and the equation for β_j shown in the “**Math**” appendix at the end of this working paper. Then nth Order Fixed Memory Polynomial **Velocity(T+1)** is constructed from the equation shown in the “**Math**” appendix.

Due to polynomial mathematics, the Velocity of the 2nd, 3rd and 4th order degree polynomial curve changes faster than it’s corresponding first order degree polynomial velocity. Whether higher order polynomial velocities are an advantage or not, will be determined by the computer when we use a walk forward optimization technique described below.

At each bar we calculate the nth order degree (1st through 4th) fixed memory polynomial velocity from the formulas in the “**Math**” appendix. As will be shown below, walk forward optimization will determine the **degree** for the nth order polynomial velocity, the number of lookback prices, **N**, needed to compute the polynomial coefficients and the threshold amounts **vup** and **vdn**. When the nth order degree velocity is greater than the threshold amount **vup** we will go long. When the velocity is less than the threshold amount **-vdn** we will go short.

Buy Rule:

IF Velocity is greater than the threshold amount **vup** then buy at the market.

Sell Rule:

IF Velocity is less than the threshold amount **-vdn** then sell at the market.

Intraday Bars Exit Rule:

Close the position at 1500 CST when the U.S stock market trading stops. (no trades will be carried overnight).

Intraday Bars First Trade of Day Entry Rule:

All trade signals before 30 minutes after the U.S. stock market opens are ignored. We've included this rule because with overnight trading there are often gaps in the open creating immediate strategy buys and sells. Many times these gaps are closed creating a losing whipsaw trade. To avoid the opening gap whipsaw trade problem, we've delayed the first trade of the day for 30 minutes after the opening until 9:00am CST

Testing The Polynomial Velocity Strategy Using Walk Forward Optimization

There will be four strategy parameters to determine:

1. **degree**, degree=1 for straight line velocity, degree=2 for 2nd order velocity, etc.
2. **N**, is the number of lookback bars of prices to calculate the **velocity**.
3. **vup**, the threshold amount that velocity must be greater than to issue a buy signal
4. **vdn**, the threshold amount that velocity must be less than to issue a sell signal

To test this strategy, we will use one minute bar prices of the NQ futures contract traded on the CME/Globex and known by the symbol NQ from January 3, 2014 to January 6, 2017.

We will test this strategy with the above NQ 1 min bars on a walk forward basis, as will be described below. To create our walk forward files we will use the **add-in** software product called the Power Walk Forward Optimizer (PWFO). In TradeStation (TS), we will run the PWFO strategy **add-in** along with the nth Order Polynomial Velocity Strategy on the NQ 1min data from January 3, 2014 to January 6, 2017. The PWFO will breakup and create 30 calendar day in-sample sections along with their corresponding one calendar week out-of-sample sections from the 162 weeks of NQ (see Walk forward Testing below) creating 158 out-of-sample weeks.

What Is An In-Sample Section and Out-Of-Sample Section?

Whenever we do a TS optimization on many different strategy inputs, TS generates a report of performance metrics (total net profits, number of losing trades, etc) vs these different inputs. If the report is sorted on say the total net profits(**tnp**) performance metric column, then the highest **tnp** would correspond to a certain set of inputs. This is called an **in-sample** or **test section**. If we choose a set of strategy inputs from this report based upon some performance metric we have no idea whether these strategy inputs will produce the same results on future price data or data they have not been tested on. Price data that is not in the **in-sample** section is defined as **out-of-sample data**. Since the good performance metrics generated in the in-sample section are mostly due to "curve fitting" (see Walk Forward Out-of-Sample Testing section below) it is important to see how the strategy inputs chosen from the in-sample section perform on out-of-sample data.

What Does The Power Walk Forward Optimizer (PWFO) Do?

The PWFO is a TS **add-in** that breaks up the TS optimization run into many user selectable in-sample and out-of-sample sections. The PWFO prints out the in-sample sample performance metrics **and the out-of-sample performance results**, on one line, for each case or input variable combination that is run by the TradeStation(TS) optimization module to a user selected spreadsheet comma delimited file. The PWFO can generate up to 500 different in-sample and out-of-sample date optimization files in one TS run, saving the user from having to generate optimization runs one at a time. The PWFO output allows you to quickly determine whether your procedure for selecting input parameters for your strategy just curve fits the price and noise, or produces statistically valid out-of-sample results. In addition to the out-of-sample performance results presented for each case, 30+ superior and robust performance metrics (many are new and never presented before) are added to each case line in the in-sample section and printed out to the comma delimited file. These 30+ performance metrics allow for a superior and robust selection of input variables from the in-sample section that have a higher probability of performing well on out-of-sample data (Please see <http://meversanalytics.com/Walk-Forward-Optimization.html> for a listing of these performance metrics).

For our computer run we will have the PWFO breakup the 162 weeks of NQ one minute bar price data into 158 in-sample and out-of-sample files. The in-sample sections will be 30 calendar days and the out-of-sample(OOS)

section will be the one week following the in-sample section. The OOS week will always end on a Friday as will the 30 calendar day in-sample section. As an example the first in-sample section would be from 11/28/2013 to 12/27/2013 and the out-of-sample section would be the week following from 12/30/2013 to 1/3/2014. (our in-sample and out-of-sample sections always end on a Friday). We would then move everything ahead a week and the 2nd in-sample section would be from 12/5/2013 to 1/3/2014 and the week following out-of-sample section would be from 1/6/2014 to 1/10/2014. Etc. Note: The In-Sample sections also end on a Friday and the OOS sections run from Monday to Friday.

The PWFO 158 in-sample/out-of-sample section dates are shown in **Table 1** below. We will then use another software product called the Walk Forward Performance Metric Explorer (WFME) on each of the 158 in-sample and out-of-sample sections generated by the PWFO to find the best in-sample section performance *filter* that determines the strategy input parameters (*degree, N, vup, vdn*) that will be used on the out-of-sample data. Detailed information about the WFME can be found at <http://meversanalytics.com/wfme.html>

For the in-sample data we will run the TradeStation optimization engine on the 158 weeks of NQ 1 min bars with the following ranges for the nth order fading memory polynomial velocity strategy input variables.

1. **pw=degree from 1 to 4**
2. **N from 20 to 70 in steps of 10.**
3. **vup from 0.25 to 3.5 steps of 0.25**
4. **vdn from 0.25 to 3.5 in steps of 0.25**
5. **Mult = 3.87, iNorm=1 (See Appendix 3, the Normalization Multiplier)**

This will produce 4704 different cases or combinations of the input parameters for each of the 158 PWFO output files.

Walk Forward Out-of-Sample Testing

Walk forward analysis attempts to minimize the curve fitting of price noise by using the law of averages from the Central Limit Theorem on the out-of-sample performance. In walk forward analysis the data is broken up into many in-sample and out-of-sample sections. Usually for any strategy, one has some performance metric selection procedure, which we will call a filter, used to select the input parameters from the in-sample optimization run. For instance, a *filter* might be all cases that have a profit factor (PF) greater than 1 and less than 3. For the number of cases left, we might select the cases that had the best percent profit. This procedure would leave you with one case in the in-sample section output and its associated strategy input parameters. Now suppose we ran our optimization on each of our many in-sample sections and applied our *filter* to each in-sample section. We would then use the strategy input parameters found by the *filter* in each in-sample section on the out-of-sample section immediately following that in-sample section. The strategy inputs found by the *filter* in each in-sample section and applied to each out-of-sample section would produce independent net profits and losses for each of the out-of-sample sections. Using this method over "x" in-sample sections, we now have "x" number of independent out-of-sample section profits and losses from the strategy inputs found by the *filter* in the in-sample sections. If we take the average of these out-of-sample section net profits and losses, then we will have an estimate of how our strategy will perform on average. Due to the Central Limit Theorem, as our sample size increases, the spurious noise results in the out-of-sample section performance tend to average out to zero in the limit leaving us with what to expect from our strategy and filter. *Mathematical note: This assumption assumes that the out-of-sample returns are from probability distributions that have a finite variance.*

Why use the walk forward technique? Why not just perform an optimization on the whole price series and choose the input parameters that give the best total net profits or profit factor or some other performance metrics? Surely the price noise cancels itself out with such a large number of in-sample prices and trades. Unfortunately, nothing could be farther from the truth! Optimization is a misnomer and should really be called combinatorial search. As stated above, whenever we run a combinatorial search over many different combinations of strategy input parameters on noisy data on a fixed number of prices, *no matter how many*, the best performance parameters found are guaranteed to be due to "*curve fitting*" the noise and signal. What do we mean by "*curve fitting*"? The price series that we trade consists of random spurious price movements, which we call noise, and repeatable price patterns (*if they exist*). When we run, for example, 5000 different input parameter combinations, the best performance parameters will be from those strategy input variables that are able to produce profits from the price pattern *and* the random spurious

movements. While the price patterns will repeat, the same spurious price movements will not. If the spurious movements that were captured by a certain set of input parameters were a large part of the total net profits, then choosing these input parameters will produce losses when traded on future data. These losses occur because the spurious movements will not be repeated in the same way. This is why strategy optimization, neural net optimizations or combinatorial searches with no out-of-sample testing cause losses when traded in real time by mistaking chance fluctuations for genuine effects. Unfortunately, it is human nature to extrapolate past performance to project future trading results and thus results from the chance fluctuations of curve fitting give the illusion, a modern “siren call” so to speak, of future trading profits.

To gain confidence that our input parameter selection method using the optimization output of the in-sample data will produce profits, we must test the input parameters we found in the in-sample section on out-of-sample data. In addition, we must perform the in-sample/out-of-sample analysis many times. Why not just do the out-of-sample analysis once or just three times? Well just as in Poker or any card game, where there is considerable variation in luck from hand to hand, walk forward out-of-sample analysis give considerable variation in week to week out-of-sample profit “luck”. That is, by pure chance we may have chosen some input parameter set that did well in the in-sample section data *and* the out-of-sample section data. To minimize this type of “luck”, statistically, we must repeat the walk forward out-of-sample (OOS) analysis over many in-sample/OOS sections and take an average of our weekly results over all out-of-sample sections. This average gives us an expected weekly return and a standard deviation of weekly returns which allows us to statistically estimate the expected equity and its path ranges for N weeks in the future.

Finding The Strategy Input Parameters in The Walk Forward Test Sections

The PWFO generates a number of performance metrics in the in-sample section. (Please see <http://meversanalytics.com/Walk-Forward-Optimization.html> for a listing of these performance metrics). The question we are attempting to answer statistically, is which performance metric or combination of performance metrics (which we will call a *filter*) in the in-sample section will produce strategy inputs that produce statistically valid profits in the out-of-sample section. In other words, we wish to find a performance metric *filter* that we can apply to the in-sample section that can give us strategy inputs that will produce, *on average*, good trading results in the out-of-sample sections. The PWFO produces a total of 32 different performance metrics in the in-sample section. If we have 4704 different input variations or cases, then the in-sample section consists of 32 columns of performance metrics for each of the 4704 different strategy inputs or rows.

An example of a simple filter would be to choose the row in the in-sample section that had the highest net profit or perhaps a row that had one the best performance metric from one of the other 32 PWFO metrics. Unfortunately, it was found that this type of simple filter very rarely produces good out-of-sample results. More complicated metric filters can produce good out-of-sample results minimizing spurious price movement biases in the selection of strategy inputs.

Here is a combination filter, found by the Walk Forward Performance Metric Explorer (WFME) (**figure 3**) that is used in this paper with good out-of-sample results. High profit factors (**PF**) in the in-sample section usually mean poor performance in the out-of-sample-section. This is a kind of reversion to the mean. So in the in-sample section we eliminate all strategy input rows that have a **PF**>**1.5**. In addition, we would like the strategy to trade a minimum 10 times a month to eliminate those strategy inputs that very rarely trade (**nt**>**10**). The PWFO generates the metric **eq2V**. This metric is the **Velocity Of A Least Squares 2nd Order Polynomial Line Fitted To The Equity Curve**. This is a measure of how fast the equity curve is moving on the last trade. Let us choose the 10 rows in the in-sample section that contain the highest(top) number of **eq2V** from the rows that are left after the **PF** and **nt** row elimination. This partial filter will now leave 10 cases or rows in the in-sample section that satisfy the above filter conditions. Suppose for this filter, within the 10 in-sample rows that are left, we want the row that has the maximum metric **mTrd** in the in-sample section. **mTrd** is the median of all trades in the In-Sample section for a given set of strategy inputs. This would produce a filter named **t10eq2V|p<1.5|nt>10|-mTrd**. This in-sample filter leaves only one row in the PWFO in-sample section with its associated strategy inputs and out-of-sample net profit in the out-of-sample section. This particular **t10eq2V|p<1.5|nt>10|-mTrd** filter finds the strategy inputs parameters in each of the 158 in-sample sections and applies these inputs to each of the 158 out-of-sample sections. Using the filter in-sample strategy inputs on the 158 out-of-sample sections, the average out-of-sample performance is calculated. In addition,

many other important out-of-sample performance statistics for this filter are calculated and summarized. **Figure 3** shows such a filter computer run along with a small sample of other filter combinations that are constructed in a similar manner. **Row 5** of the sample output in **Figure 3** shows the results of the filter discussed above. A total of 57600 different metric filters were examined. We chose **Row 5** because it had a lower **Dev²** and a higher **eqR²** along with better statistics than the rows above and below it. More on this below and on how that number of filters combinations effect the probability that the filter chosen was or was not due to chance

Bootstrap Probability of Filter Results: Using modern "Bootstrap" techniques, we can calculate the probability of obtaining each filter's total out-of-sample *net* profits by chance. By *net* we mean subtracting the cost and slippage of all round trip trades from the total out-of-sample profits. Here is how the bootstrap technique is applied. Suppose as an example, we calculate the total out-of-sample net profits(tOnpNet) over all out-of-sample weeks for a given filter like above. A mirror filter is created. However, instead of picking an out-of-sample net profit(OSNP) from a row that the filter picks, the mirror filter picks a *random* row's OSNP in each of the 158 PWFO files. Suppose we repeat this random row section 5000 times. Each of the 5000 mirror filters will choose a random row's OSNP of their own in each of the 158 PWFO files. At the end, each of the 5000 mirror filters will have 158 *random* OSNP's picked from the rows of the 158 PWFO files. The sum of the 158 random OSNP picks for each of the 5000 mirror filters will generate a random total out-of-sample net profit(tOnpNet). The average and standard deviation of the 5000 mirror filter's different random tOnpNets will allow us to calculate the chance probability for each of *our* filter's tOnpNet. Thus given the mirror filter's bootstrap random tOnpNet average and standard deviation, we can calculate the probability of obtaining our filter's tOnpNet by pure chance alone. Since for this run we examined 57600(shown in Figure 3) different filters, we can calculate the expected number of cases that we could obtain by pure chance that would match or exceed the tOnpNet of the filter we have chosen or $1-(1-tOnpNet\ Probability)^{57600} \sim 57600 * tOnpNet\ Probability$. For our filter in row 5 in Figure 3 the expected number of cases that we could obtain by pure chance that would match or exceed the tOnpNet of \$34,335 of Row 5 is $57660 \times 3.2 \times 10^{-6} = 0.184$. This is much less than one case so it is improbable that our result was due to pure chance.

Results

Table 1 below presents a table of the 158 in-sample and out-of-sample windows, the selected optimum parameters and the weekly out-of-sample results using the filter described above.

Figure 1 presents a graph of the equity and net equity curves generated by using the filter on the 158 weeks ending 1/3/14 to 1/6/17. The equity curves are plotted from the Equity and Net Equity columns in Table 1. Plotted on the equity curves are 2nd Order Polynomial fits. The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity. The brown line is the net equity curve with commissions and the green dots are the new highs in net equity. The grey line is the weekly NQ prices superimposed on the equity chart.

Figure 2 Walk Forward Out-Of-Sample Performance for NQ nth Order Fixed Memory Polynomial Velocity Strategy **1** minute bar chart of NQ from 3/10/2016

Figure 3 Partial output of the Walk Forward Metric Performance Explorer (WFME)
Run on the 158 PWFO files of the NQ 1min bars Nth Order Fixed Memory Polynomial Velocity

Discussion of Strategy Performance

In Figure 3 Row 5 of the spreadsheet filter output are some statistics that are of interest for our filter. **BE** is the break even weeks. Assuming the trade average and standard deviation for this filter are from a normal distribution, this is how many weeks we need to trade this strategy so that we have a 98% probability that the equity paths after that number of weeks will be greater than zero. BE is 36.8 weeks for this filter. This means we would have to trade this strategy for at least 36.8 weeks to have a 98% probability that our equity would be positive. Another interesting statistic is **Blw**. Blw is the maximum number of weeks the OSNP equity curve failed to make a new high. Blw is 12 weeks for this filter. This means that 12 weeks was the longest time that the equity for this strategy failed to make a new equity high.

To see the effect of walk forward analysis, look at **Table 1**. Notice how the input parameters *pw*, *N*, *vup* and *vdn* take sudden jumps from high to low and back. This is the walk forward process quickly adapting to changing volatility conditions in the in-sample sample. In addition, notice how often *degree* changes from a straight line velocity with *degree=1* to a 2nd, 3rd and 4th order velocity with *degree= 2, 3 and 4*. The 3rd and 4th order velocities, due to the higher order components, change much faster than the straight line velocity. When the data gets very noisy with a lot of spurious price movements, it's better to have the velocity change slower filtering out the noisy data. During other times when the noise level is not as much it is better to have the velocity break its *vup* and *vdn* barriers faster to get onboard a trend faster. This is what the filter is doing. When there is a lot of noise in the in-sample section it switches to the 1st or 2nd order curve velocity. When the noise level is lower in the in-sample section, it switches to the faster changing 3rd or 4th order curve velocity.

Using this filter, the strategy was able to generate \$34,335 net equity after commissions and slippage trading one NQ contract for 158 weeks. Note \$20 roundtrip commission and slippage was subtracted from each trade and no positions were carried overnight. The largest trade loss was -\$2000, the largest losing week was -\$3030 and the largest drawdown was -\$4490. The longest time between new equity highs was 12 weeks.

In observing Table 1 we can see that this strategy and filter made trades from a low of no trades/week to a high of 7 trades/week with an average of 2 trades/week on the weeks it did trade. The strategy seemed to wait for really strong trends and then initiate a buy or sell. There were many weeks that had no trades. Out of the 158 out-of-sample weeks the filter only traded 140 of those weeks or 89% of the time with 52% of all trades profitable. In observing the Equity Curve plot in Figure 1 we can see that the equity did quite well in both big up and down moves of the NQ. In observing the chart from 12/4/15 to 2/12/16 where NQ declined from 4684 to 3983 we can see the strategy did well on this unusually large NQ decline. After the Brexit vote on 6/23/2016, the strategy did well in the volatile market from Friday 6/24 to Friday 7/1 making \$2035 on 5 trades that week.

References

1. Efron, B., Tibshirani, R.J., (1993), "An Introduction to the Bootstrap", New York, Chapman & Hall/CRC.
2. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.

**Figure 1 Graph of Net Equity Curve Applying the Walk Forward Filter Each Week
On NQ 1min Bar Prices 01/03/14 – 01/06/17**

Note: The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity. The brown line is the equity curve with commissions and the green dots are the new highs in net equity. The grey line is the NQ Weekly Closing prices superimposed on the Equity Chart.

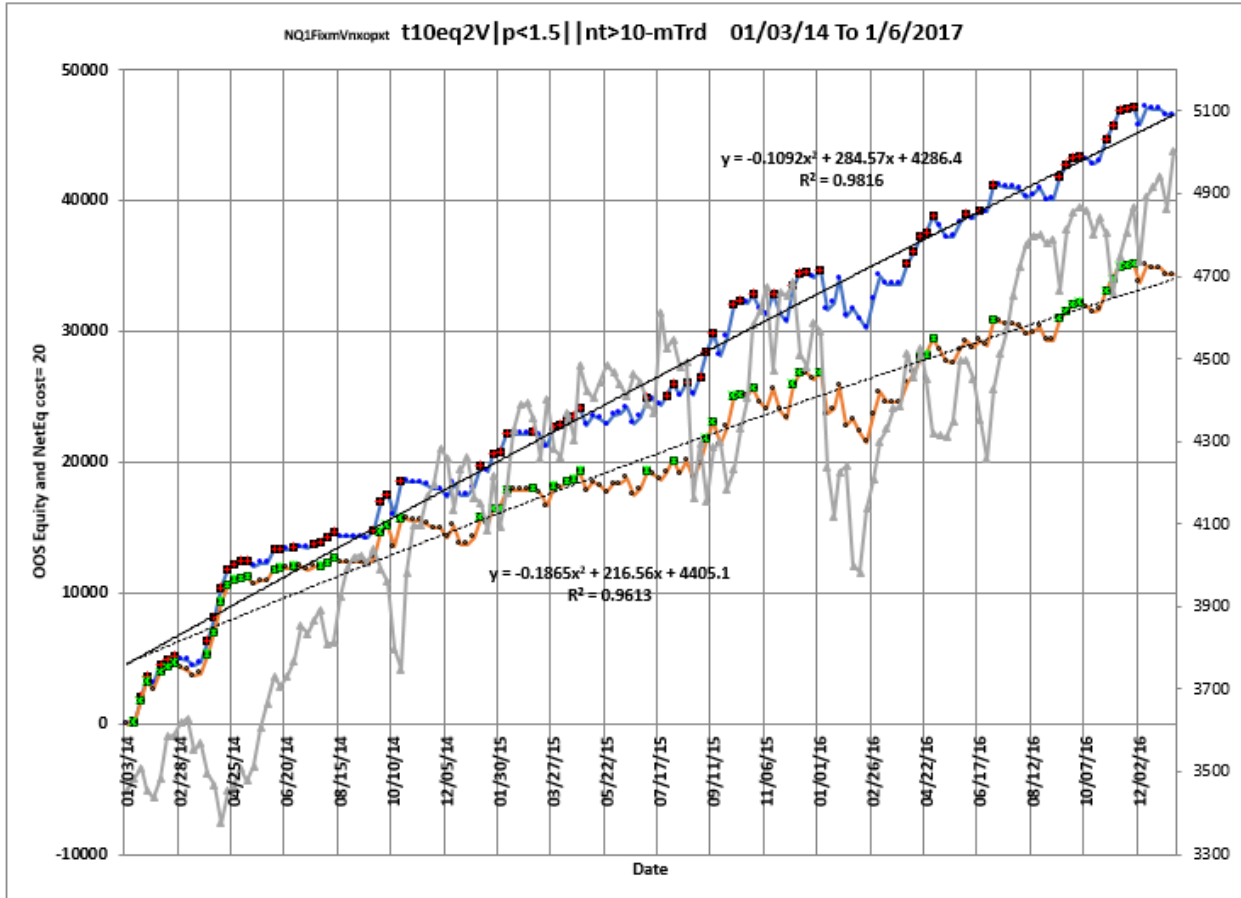
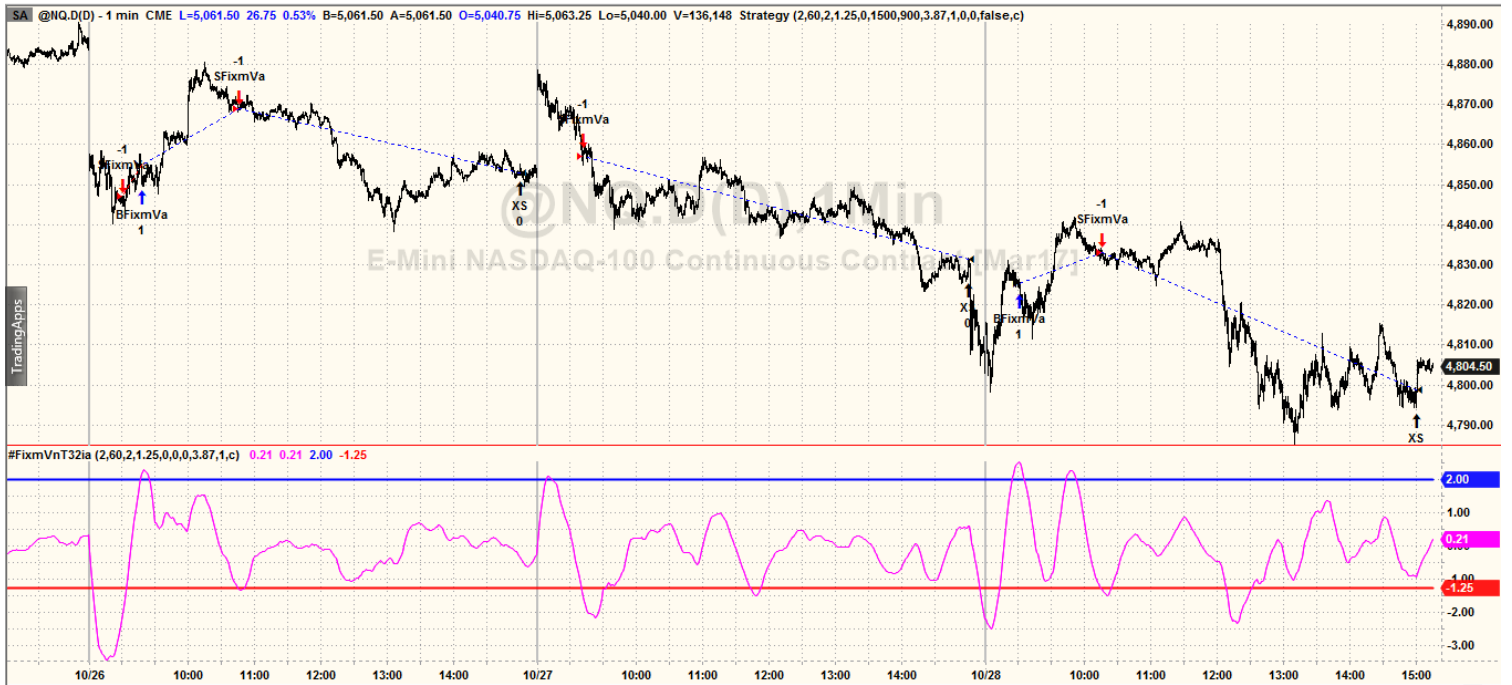


Figure 2 Walk Forward Out-Of-Sample Performance Summary for Nth Order Fixed Memory Polynomial Velocity Strategy NQ 1 minute bar chart from 10/28/16



**Figure 3_ Partial output of the Walk Forward Metric Performance Explorer (WFME)
NQ1 min bars Nth Order Fixed Memory Velocity Strategy**

1	NQ1FixmVnxopxt	s01/03/14	e01/06/17	#158	AnyTnp					a(193.9)	s97.3	f57660									c=\$20	
2	Filter-Metric	toGP	aoGP	aoTr	ao#T	oW oL	%Wtr	t	std	LLtr	LLp	eqDD	LRp	#	eqTrn	eqV^2	eqR2	Dev^2	Blw	BE	tOnpNet	Prob
3	t10PF p<1.5 nt>10-mTrd	55935	361	63.9	5.7	1.32	52	3.59	1251	-1475	-3475	-5130	7	155	386	529	97	2863	22	48.1	38415	2.82E-06
4	t20eq10 p<2 l3 nt>10-eq2A	46855	358	97.0	3.7	1.48	52	4.34	943	-1600	-1825	-2820	4	131	308	253	96	2915	10	27.8	37195	4.54E-07
5	t10eq2V p<1.5 nt>10-mTrd	46555	333	76.2	4.4	1.38	52	3.9	1009	-2000	-3030	-4490	4	140	267	250	98	1682	12	36.8	34335	3.20E-06
6	t10eq2V p<1.5 l5 nt>10-mTrd	46420	334	76.0	4.4	1.36	52	3.96	993	-2000	-3030	-4490	4	139	280	211	98	1649	12	35.4	34200	3.08E-06
7	t20mWb mLb p<1.5 l3 nt>10-m(p-rd)	49875	378	63.3	6	1.41	51	3.77	1152	-1560	-3755	-4040	4	132	328	335	98	1998	11	37.2	34115	1.67E-06
8	t20eq2V p<2 l3 nt>10-eq2A	43355	331	90.5	3.7	1.46	52	4.1	924	-1600	-1825	-2820	4	131	282	234	95	2811	14	31.2	33775	1.72E-06
9	t20%P p<1.5 l5 nt>10-PF	49965	329	60.7	5.4	1.42	50	4.14	979	-1665	-2235	-3010	4	152	298	400	97	2292	12	35.5	33505	1.03E-05

The WFME Filter Output Columns are defined as follows:

Row 1 NQ1FixmVnxopxt is the strategy abbreviation, First OOS Week End Date(1/3/14), Last OOS Week End Date(1/6/17), **Number of weeks(#158)** a=weekly net average of bootstrap random picks. s= weekly standard deviation of bootstrap random picks. f=number of different filters examined. c= slippage and round trip trade cost(c=\$20).

Filter = The filter that was run. Row10 filter **t10eq2V|p<1.5|nt>10|-mTrd**

The t10eq2V|p<1.5|nt>10|-mTrd filter produced the following average 158 week statistics on Row 7.

toGP = Total out-of-sample(OOS) gross profit for these 158 weeks.

aoGP = Average OOS gross profit for the 158 weeks

aoTr = Average OOS profit per trade

ao#T = Average number of OOS trades per week

oW|oL = The average OOS winning trade divided by the average OOS losing trade

%Wtr = percent of all OOS trades that were winning trades

t = The student t statistic for the 158 weekly OOS profits. The higher the t statistic the higher the probability that this result was not due to pure chance

std = The standard deviation of the 158 weekly OOS profits

LLtr = The largest losing OOS trade in all OOS periods,

LLp = The largest losing OOS period(week)

eqDD = The OOS equity drawdown

LRp = The largest number of losing OOS weeks in a row

= The number of weeks this filter produced a weekly result. Note for some weeks there can be no strategy inputs that satisfy a given filter's criteria.

eqTrn = The straight line trend of the OOS gross profit equity curve in \$/week.

eqV^2 = The ending velocity of 2nd order polynomial that is fit to the OOS equity curve

eqR2 = The correlation coefficient(r^2) of a straight line fit to the OOS equity curve

Dev² = A measure of equity curve smoothness. The square root of the average [(equity curve minus a straight line)²]

Blw = The maximum number of weeks the OOS equity curve failed to make a new high.

BE = Break even weeks. Assuming the average and standard deviation are from a normal distribution, this is the number of weeks you would have to trade to have a 98% probability that your OOS equity is above zero.

tOnpNet = Total out-of-sample net profit(tOnpNet) minus the total trade cost.
$$tOnpNet = tOnp - (\text{Number of trade weeks}) * aOnT * \text{Cost}$$

Prob = the probability that the filter's tOnpNet was due to pure chance.

Table 1 Walk Forward Out-Of-Sample Performance Summary for NQ1min Nth Order Fixed Memory Polynomial Velocity Strategy

NQ-1 min bars 1/3/2014 - 1/6/2017. The input values *degree(pw)*, *N*, *vup*, *vdn* are the values found from applying the filter to the in-sample section optimization runs.

Filter = $t10eq2V|p<1.5|nt>10|-mTrd$, $PF<=1.5$, $nt>=10$ then top 10 $eq2V$ and then maximum $mTrd$

osnp = Weekly Out-of-sample gross profit in \$

Equity = Running Sum of weekly out-of-sample gross profits \$

NOnp\$20 = Weekly Out-Of-Sample Net Profit in \$ = **osnp-ont*20**.

NetEq = running sum of the weekly out-of-sample net profits in \$

ollt = The largest losing trade in the out-of-sample section in \$.

odd = The drawdown in the out-of-sample section in \$.

ont = The number of trades in the out-of-sample week.

pw= degree, degree=1 for straight line velocity, degree=2 for 2nd order velocity, etc.

N = N the lookback period

vup, the threshold amount that velocity has to be greater than to issue a buy signal

vdn, the threshold amount that velocity has to be less than to issue a sell signal

Note: Blank rows indicate that no out-of-sample trades were made that week

In-Sample Dates			Out-Of-Sample Dates			osnp	Eq	NOnp\$20	NetEq	ont	ollt	odd	pw	N	vup	vdn
11/28/13	To	12/27/13	12/30/13	To	01/03/14	-45	-45	-85	-85	2	-75	-75	1	70	1.25	1
12/05/13	To	01/03/14	01/06/14	To	01/10/14	260	215	180	95	4	-220	-260	4	30	1	2.5
12/12/13	To	01/10/14	01/13/14	To	01/17/14	1820	2035	1720	1815	5	-95	-95	2	20	1	1.5
12/19/13	To	01/17/14	01/20/14	To	01/24/14	1505	3540	1405	3220	5	-180	-180	3	40	1.5	1
12/26/13	To	01/24/14	01/27/14	To	01/31/14	-505	3035	-605	2615	5	-690	-925	3	30	1	2.75
01/02/14	To	01/31/14	02/03/14	To	02/07/14	1480	4515	1360	3975	6	-750	-750	4	20	2.5	1.25
01/09/14	To	02/07/14	02/10/14	To	02/14/14	430	4945	410	4385	1	0	0	2	70	1.75	2.25
01/16/14	To	02/14/14	02/17/14	To	02/21/14	270	5215	230	4615	2	-15	-15	2	60	1.25	3
01/23/14	To	02/21/14	02/24/14	To	02/28/14	-275	4940	-355	4260	4	-370	-475	2	60	1.25	3
01/30/14	To	02/28/14	03/03/14	To	03/07/14	-95	4845	-155	4105	3	-115	-115	4	40	1.75	3
02/06/14	To	03/07/14	03/10/14	To	03/14/14	-420	4425	-500	3605	4	-545	-585	4	50	1.5	2.5
02/13/14	To	03/14/14	03/17/14	To	03/21/14	275	4700	195	3800	4	-115	-115	4	50	2.75	1.5
02/20/14	To	03/21/14	03/24/14	To	03/28/14	1590	6290	1470	5270	6	-220	-220	4	50	3	1.5
02/27/14	To	03/28/14	03/31/14	To	04/04/14	1795	8085	1735	7005	3	-245	-245	2	70	2.75	1.5
03/06/14	To	04/04/14	04/07/14	To	04/11/14	2330	10415	2290	9295	2	0	0	1	70	3	1.75
03/13/14	To	04/11/14	04/14/14	To	04/18/14	1380	11795	1320	10615	3	0	0	3	20	2.25	2.75
03/20/14	To	04/18/14	04/21/14	To	04/25/14	405	12200	385	11000	1	0	0	3	20	2.25	3
03/27/14	To	04/25/14	04/28/14	To	05/02/14	220	12420	160	11160	3	-305	-305	4	30	2.25	3.5
04/03/14	To	05/02/14	05/05/14	To	05/09/14	75	12495	55	11215	1	0	0	4	30	2.25	3.5
04/10/14	To	05/09/14	05/12/14	To	05/16/14	-500	11995	-560	10655	3	-330	-545	3	40	1.5	2.5
04/17/14	To	05/16/14	05/19/14	To	05/23/14	265	12260	245	10900	1	0	0	1	40	1.75	1.75
04/24/14	To	05/23/14	05/26/14	To	05/30/14		12260		10900				2	70	1.75	2
05/01/14	To	05/30/14	06/02/14	To	06/06/14	1025	13285	945	11845	4	-20	-20	2	60	0.5	3.25
05/08/14	To	06/06/14	06/09/14	To	06/13/14	60	13345	20	11865	2	0	0	3	60	1.25	1.75
05/15/14	To	06/13/14	06/16/14	To	06/20/14		13345		11865				2	30	1.25	1.75
05/22/14	To	06/20/14	06/23/14	To	06/27/14	195	13540	135	12000	3	-530	-530	3	50	1.5	1.25
05/29/14	To	06/27/14	06/30/14	To	07/04/14		13540		12000				3	70	1.5	1.25
06/05/14	To	07/04/14	07/07/14	To	07/11/14	-115	13425	-275	11725	8	-480	-795	4	50	1.75	1.25
06/12/14	To	07/11/14	07/14/14	To	07/18/14	345	13770	265	11990	4	-255	-255	2	70	1	2.5
06/19/14	To	07/18/14	07/21/14	To	07/25/14	120	13890	80	12070	2	0	0	1	60	1.25	1.25
06/26/14	To	07/25/14	07/28/14	To	08/01/14	395	14285	295	12365	5	-390	-390	1	50	1.75	1.25
07/03/14	To	08/01/14	08/04/14	To	08/08/14	395	14680	355	12720	2	-60	-60	4	70	3	1.75
07/10/14	To	08/08/14	08/11/14	To	08/15/14	-365	14315	-385	12335	1	-365	-365	1	50	2.5	1.25

In-Sample Dates			Out-Of-Sample Dates			osnp	Eq	NOnp\$20	NetEq	ont	ollt	odd	pw	N	vup	vdn
07/17/14	To	08/15/14	08/18/14	To	08/22/14		14315		12335				3	70	1.5	3
07/24/14	To	08/22/14	08/25/14	To	08/29/14		14315		12335				4	50	2	3
07/31/14	To	08/29/14	09/01/14	To	09/05/14		14315		12335				4	50	2	3
08/07/14	To	09/05/14	09/08/14	To	09/12/14	-135	14180	-215	12120	4	-615	-615	4	50	1.25	2.5
08/14/14	To	09/12/14	09/15/14	To	09/19/14	660	14840	560	12680	5	-200	-200	4	50	1	2.5
08/21/14	To	09/19/14	09/22/14	To	09/26/14	2105	16945	1985	14665	6	-240	-240	1	30	1	1.75
08/28/14	To	09/26/14	09/29/14	To	10/03/14	575	17520	475	15140	5	-650	-650	2	60	1.25	3.25
09/04/14	To	10/03/14	10/06/14	To	10/10/14	-1505	16015	-1645	13495	7	-1080	-2485	2	60	1.5	3.25
09/11/14	To	10/10/14	10/13/14	To	10/17/14	2525	18540	2165	15660	18	-545	-785	1	30	2	1
09/18/14	To	10/17/14	10/20/14	To	10/24/14		18540		15660				4	40	3	3.25
09/25/14	To	10/24/14	10/27/14	To	10/31/14	-115	18425	-155	15505	2	-105	-115	4	40	3	3.25
10/02/14	To	10/31/14	11/03/14	To	11/07/14		18425		15505				4	40	3	3.25
10/09/14	To	11/07/14	11/10/14	To	11/14/14	-185	18240	-205	15300	1	-185	-185	1	50	1.75	3.5
10/16/14	To	11/14/14	11/17/14	To	11/21/14	-290	17950	-350	14950	3	-215	-290	2	20	1.25	2.5
10/23/14	To	11/21/14	11/24/14	To	11/28/14		17950		14950				2	40	1.5	2
10/30/14	To	11/28/14	12/01/14	To	12/05/14	-590	17360	-690	14260	5	-355	-715	2	60	1.25	2.25
11/06/14	To	12/05/14	12/08/14	To	12/12/14	1005	18365	885	15145	6	-1355	-1615	2	40	2.5	1
11/13/14	To	12/12/14	12/15/14	To	12/19/14	-870	17495	-1470	13675	30	-810	-1740	4	20	2.75	1.75
11/20/14	To	12/19/14	12/22/14	To	12/26/14		17495		13675				1	60	3.25	1.25
11/27/14	To	12/26/14	12/29/14	To	01/02/15	570	18065	530	14205	2	0	0	2	60	3.5	1.75
12/04/14	To	01/02/15	01/05/15	To	01/09/15	1685	19750	1585	15790	5	-145	-145	1	40	3.25	2.25
12/11/14	To	01/09/15	01/12/15	To	01/16/15	-460	19290	-540	15250	4	-1315	-1315	1	60	2.5	2.25
12/18/14	To	01/16/15	01/19/15	To	01/23/15	1310	20600	1210	16460	5	-570	-570	3	40	2.75	2.25
12/25/14	To	01/23/15	01/26/15	To	01/30/15	125	20725	45	16505	4	-735	-1020	3	30	2.25	3.25
01/01/15	To	01/30/15	02/02/15	To	02/06/15	1410	22135	1370	17875	2	-65	-65	2	30	2.5	3.25
01/08/15	To	02/06/15	02/09/15	To	02/13/15		22135		17875				4	60	2.5	3.5
01/15/15	To	02/13/15	02/16/15	To	02/20/15		22135		17875				4	70	3	3.25
01/22/15	To	02/20/15	02/23/15	To	02/27/15		22135		17875				1	50	2	3.5
01/29/15	To	02/27/15	03/02/15	To	03/06/15	200	22335	140	18015	3	-340	-380	1	50	2	1.75
02/05/15	To	03/06/15	03/09/15	To	03/13/15	-295	22040	-395	17620	5	-400	-650	3	70	1.5	1.75
02/12/15	To	03/13/15	03/16/15	To	03/20/15	-850	21190	-1010	16610	8	-435	-960	4	40	2.5	1.25
02/19/15	To	03/20/15	03/23/15	To	03/27/15	1585	22775	1505	18115	4	-375	-375	4	60	3.25	2
02/26/15	To	03/27/15	03/30/15	To	04/03/15	10	22785	-70	18045	4	-220	-220	4	60	2.25	2
03/05/15	To	04/03/15	04/06/15	To	04/10/15	510	23295	470	18515	2	0	0	3	50	2	3
03/12/15	To	04/10/15	04/13/15	To	04/17/15	215	23510	175	18690	2	-90	-90	4	30	1.5	3.25
03/19/15	To	04/17/15	04/20/15	To	04/24/15	650	24160	590	19280	3	0	0	4	50	1.75	3
03/26/15	To	04/24/15	04/27/15	To	05/01/15	-1370	22790	-1490	17790	6	-825	-2060	1	70	0.25	2.5
04/02/15	To	05/01/15	05/04/15	To	05/08/15	720	23510	640	18430	4	-50	-70	2	70	2.25	1.75
04/09/15	To	05/08/15	05/11/15	To	05/15/15	-160	23350	-240	18190	4	-515	-910	1	50	2	2
04/16/15	To	05/15/15	05/18/15	To	05/22/15	-500	22850	-520	17670	1	-500	-500	2	40	1.75	2.75
04/23/15	To	05/22/15	05/25/15	To	05/29/15	730	23580	650	18320	4	-65	-65	2	40	1	2.75
04/30/15	To	05/29/15	06/01/15	To	06/05/15	110	23690	30	18350	4	-115	-120	2	70	1.5	3.25
05/07/15	To	06/05/15	06/08/15	To	06/12/15	470	24160	410	18760	3	-420	-420	1	70	1.75	2
05/14/15	To	06/12/15	06/15/15	To	06/19/15	-1220	22940	-1300	17460	4	-530	-1220	3	70	2.75	1.5
05/21/15	To	06/19/15	06/22/15	To	06/26/15	545	23485	485	17945	3	-45	-45	4	60	2.75	2.25
05/28/15	To	06/26/15	06/29/15	To	07/03/15	1430	24915	1370	19315	3	-100	-100	2	70	2	1.5
06/04/15	To	07/03/15	07/06/15	To	07/10/15	-155	24760	-295	19020	7	-1070	-1955	1	50	1	2.75
06/11/15	To	07/10/15	07/13/15	To	07/17/15	-360	24400	-380	18640	1	-360	-360	2	50	2.25	2
06/18/15	To	07/17/15	07/20/15	To	07/24/15	600	25000	540	19180	3	-260	-260	2	50	2	1.75
06/25/15	To	07/24/15	07/27/15	To	07/31/15	1015	26015	935	20115	4	-135	-250	4	20	2	2.25
07/02/15	To	07/31/15	08/03/15	To	08/07/15	-945	25070	-985	19130	2	-600	-945	1	50	2.75	2.75
07/09/15	To	08/07/15	08/10/15	To	08/14/15	1015	26085	935	20065	4	-415	-415	2	70	1.75	3.5
07/16/15	To	08/14/15	08/17/15	To	08/21/15	-930	25155	-1290	18775	18	-925	-2720	4	50	2.5	2.5
07/23/15	To	08/21/15	08/24/15	To	08/28/15	1355	26510	1135	19910	11	-1275	-1835	1	70	1.25	3

In-Sample Dates			Out-Of-Sample Dates			osnp	Eq	NOnp\$20	NetEq	ont	ollt	odd	pw	N	vup	vdn
07/30/15	To	08/28/15	08/31/15	To	09/04/15	1970	28480	1870	21780	5	-690	-690	1	60	2.25	2.5
08/06/15	To	09/04/15	09/07/15	To	09/11/15	1325	29805	1265	23045	3	-475	-475	2	40	3.5	3.25
08/13/15	To	09/11/15	09/14/15	To	09/18/15	-1640	28165	-1720	21325	4	-575	-1640	4	40	3.5	3.5
08/20/15	To	09/18/15	09/21/15	To	09/25/15	1440	29605	1340	22665	5	-205	-295	4	40	3.5	3.5
08/27/15	To	09/25/15	09/28/15	To	10/02/15	2445	32050	2325	24990	6	-800	-1110	1	30	1.75	3.5
09/03/15	To	10/02/15	10/05/15	To	10/09/15	260	32310	220	25210	2	0	0	3	40	2.25	3
09/10/15	To	10/09/15	10/12/15	To	10/16/15	-110	32200	-130	25080	1	-110	-110	1	30	2.5	2.25
09/17/15	To	10/16/15	10/19/15	To	10/23/15	610	32810	570	25650	2	0	0	1	70	1.75	3.25
09/24/15	To	10/23/15	10/26/15	To	10/30/15	-1070	31740	-1090	24560	1	-1070	-1070	1	40	3	3.5
10/01/15	To	10/30/15	11/02/15	To	11/06/15	-465	31275	-525	24035	3	-465	-480	1	20	2.25	2.25
10/08/15	To	11/06/15	11/09/15	To	11/13/15	1585	32860	1545	25580	2	0	0	4	40	2.75	2.5
10/15/15	To	11/13/15	11/16/15	To	11/20/15	-1540	31320	-1620	23960	4	-1095	-1635	4	60	2.75	2
10/22/15	To	11/20/15	11/23/15	To	11/27/15	-580	30740	-660	23300	4	-540	-625	1	20	2	1.75
10/29/15	To	11/27/15	11/30/15	To	12/04/15	2725	33465	2645	25945	4	0	0	1	70	2.25	1.5
11/05/15	To	12/04/15	12/07/15	To	12/11/15	975	34440	895	26840	4	-590	-590	1	40	2.75	2.25
11/12/15	To	12/11/15	12/14/15	To	12/18/15	105	34545	-55	26785	8	-815	-1600	3	20	2.25	3.5
11/19/15	To	12/18/15	12/21/15	To	12/25/15	-415	34130	-435	26350	1	-415	-415	2	40	3	2.25
11/26/15	To	12/25/15	12/28/15	To	01/01/16	555	34685	535	26885	1	0	0	1	60	1.5	3.25
12/03/15	To	01/01/16	01/04/16	To	01/08/16	-3030	31655	-3230	23655	10	-1355	-3030	3	60	3	3
12/10/15	To	01/08/16	01/11/16	To	01/15/16	535	32190	375	24030	8	-2000	-2000	1	70	3.25	2
12/17/15	To	01/15/16	01/18/16	To	01/22/16	1840	34030	1760	25790	4	-330	-330	1	30	2.5	3.5
12/24/15	To	01/22/16	01/25/16	To	01/29/16	-2900	31130	-3060	22730	8	-1795	-4490	1	60	2.5	3.5
12/31/15	To	01/29/16	02/01/16	To	02/05/16	595	31725	535	23265	3	0	0	1	20	2.5	3.5
01/07/16	To	02/05/16	02/08/16	To	02/12/16	-785	30940	-945	22320	8	-1015	-2420	1	40	1.5	3.5
01/14/16	To	02/12/16	02/15/16	To	02/19/16	-700	30240	-780	21540	4	-590	-1065	1	20	3.5	0.5
01/21/16	To	02/19/16	02/22/16	To	02/26/16	2190	32430	2070	23610	6	-500	-500	1	70	0.5	3.5
01/28/16	To	02/26/16	02/29/16	To	03/04/16	1800	34230	1760	25370	2	-25	-25	1	60	2	1.75
02/04/16	To	03/04/16	03/07/16	To	03/11/16	-605	33625	-785	24585	9	-900	-1375	2	30	0.75	3.5
02/11/16	To	03/11/16	03/14/16	To	03/18/16		33625		24585				2	60	2.5	3
02/18/16	To	03/18/16	03/21/16	To	03/25/16		33625		24585				1	60	3	1.75
02/25/16	To	03/25/16	03/28/16	To	04/01/16	1525	35150	1465	26050	3	-185	-185	2	50	2	2.25
03/03/16	To	04/01/16	04/04/16	To	04/08/16	895	36045	815	26865	4	-700	-700	1	50	1.75	2.25
03/10/16	To	04/08/16	04/11/16	To	04/15/16	1185	37230	1125	27990	3	-210	-210	2	40	2.5	2
03/17/16	To	04/15/16	04/18/16	To	04/22/16	295	37525	235	28225	3	-115	-115	4	60	3	1.75
03/24/16	To	04/22/16	04/25/16	To	04/29/16	1300	38825	1220	29445	4	-380	-380	4	60	3.25	1.75
03/31/16	To	04/29/16	05/02/16	To	05/06/16	-785	38040	-865	28580	4	-945	-1080	1	70	2.75	1.25
04/07/16	To	05/06/16	05/09/16	To	05/13/16	-830	37210	-870	27710	2	-560	-830	3	30	1.75	3.5
04/14/16	To	05/13/16	05/16/16	To	05/20/16	5	37215	-135	27575	7	-305	-700	1	50	1.5	2
04/21/16	To	05/20/16	05/23/16	To	05/27/16	1055	38270	995	28570	3	0	0	1	30	1	3
04/28/16	To	05/27/16	05/30/16	To	06/03/16	740	39010	660	29230	4	-40	-40	1	30	1	3.25
05/05/16	To	06/03/16	06/06/16	To	06/10/16	-405	38605	-485	28745	4	-250	-525	4	70	2	2.5
05/12/16	To	06/10/16	06/13/16	To	06/17/16	660	39265	540	29285	6	-500	-715	4	60	1.5	3.25
05/19/16	To	06/17/16	06/20/16	To	06/24/16	-110	39155	-330	28955	11	-735	-770	3	60	2.25	1.5
05/26/16	To	06/24/16	06/27/16	To	07/01/16	2035	41190	1935	30890	5	-55	-55	1	60	1.75	1.75
06/02/16	To	07/01/16	07/04/16	To	07/08/16	-35	41155	-135	30755	5	-715	-810	1	70	1.5	1.75
06/09/16	To	07/08/16	07/11/16	To	07/15/16	-140	41015	-180	30575	2	-80	-140	1	40	2	3.5
06/16/16	To	07/15/16	07/18/16	To	07/22/16		41015		30575				1	50	2.5	3.5
06/23/16	To	07/22/16	07/25/16	To	07/29/16	-130	40885	-150	30425	1	-130	-130	1	40	2.25	3.5
06/30/16	To	07/29/16	08/01/16	To	08/05/16	-625	40260	-665	29760	2	-330	-625	3	60	2	1.5
07/07/16	To	08/05/16	08/08/16	To	08/12/16	190	40450	170	29930	1	0	0	4	70	1.75	2.75
07/14/16	To	08/12/16	08/15/16	To	08/19/16	525	40975	425	30355	5	-160	-210	3	60	0.75	2.75
07/21/16	To	08/19/16	08/22/16	To	08/26/16	-940	40035	-1040	29315	5	-335	-940	1	50	1.25	1.75
07/28/16	To	08/26/16	08/29/16	To	09/02/16	80	40115	0	29315	4	-340	-465	4	40	2.5	1
08/04/16	To	09/02/16	09/05/16	To	09/09/16	1735	41850	1695	31010	2	0	0	4	40	2.5	1.25

In-Sample Dates			Out-Of-Sample Dates			osnp	Eq	NOnp\$20	NetEq	ont	ollt	odd	pw	N	vup	vdn
08/11/16	To	09/09/16	09/12/16	To	09/16/16	830	42680	510	31520	16	-295	-725	2	30	1.5	1.5
08/18/16	To	09/16/16	09/19/16	To	09/23/16	600	43280	580	32100	1	0	0	2	20	2	2.5
08/25/16	To	09/23/16	09/26/16	To	09/30/16	110	43390	50	32150	3	-160	-165	1	20	2	2.5
09/01/16	To	09/30/16	10/03/16	To	10/07/16	-240	43150	-300	31850	3	-365	-470	1	20	2.25	2
09/08/16	To	10/07/16	10/10/16	To	10/14/16	-355	42795	-395	31455	2	-530	-530	3	20	3	2
09/15/16	To	10/14/16	10/17/16	To	10/21/16	260	43055	200	31655	3	-85	-85	3	20	3.25	1.75
09/22/16	To	10/21/16	10/24/16	To	10/28/16	1585	44640	1425	33080	8	-245	-400	2	60	2	1.25
09/29/16	To	10/28/16	10/31/16	To	11/04/16	1050	45690	990	34070	3	0	0	2	40	2.75	1.75
10/06/16	To	11/04/16	11/07/16	To	11/11/16	1170	46860	930	35000	12	-510	-780	1	30	2	1.5
10/13/16	To	11/11/16	11/14/16	To	11/18/16	155	47015	95	35095	3	-140	-140	1	40	1.75	3
10/20/16	To	11/18/16	11/21/16	To	11/25/16	155	47170	135	35230	1	0	0	2	50	2.25	3.25
10/27/16	To	11/25/16	11/28/16	To	12/02/16	-1390	45780	-1490	33740	5	-825	-1390	2	70	0.75	3.5
11/03/16	To	12/02/16	12/05/16	To	12/09/16	1330	47110	1290	35030	2	0	0	2	50	2.25	3.25
11/10/16	To	12/09/16	12/12/16	To	12/16/16	-105	47005	-205	34825	5	-230	-310	1	60	2	1.25
11/17/16	To	12/16/16	12/19/16	To	12/23/16		47005		34825				4	50	1.75	3.5
11/24/16	To	12/23/16	12/26/16	To	12/30/16	-450	46555	-490	34335	2	-375	-450	3	40	2	2.5
12/01/16	To	12/30/16	01/02/17	To	01/06/17		46555		34335				4	20	2.5	2.5

Appendix I: n^{th} Order Polynomial Next Bar's Forecast Math

What is the n^{th} Order Polynomial ?

The n^{th} Order Polynomial, also called the n^{th} Order Fixed Memory Polynomial, is simply the least square fit of a polynomial of the form $\mathbf{b}_0 + \mathbf{b}_1 * \mathbf{t} + \mathbf{b}_2 * \mathbf{t}^2 + \mathbf{b}_3 * \mathbf{t}^3 + \dots + \mathbf{b}_n * \mathbf{t}^n$ to a *fixed* number of past data points. Where \mathbf{t} is discrete time bars. Time could be daily bars or one minute bars. We use the term "Fixed Memory" to designate that only a fixed number of data points are used to calculate the polynomial coefficients. It is assumed that the time bars occur at fixed intervals of time so tic bars would not be appropriate for this analysis. Least squares is a mathematical technique where the squared vertical distance between the data and the curve that is being fit to the data is minimized. When the net squared distance (also called the sum of the squared errors) is minimized, a unique set of coefficients $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n$ for the polynomial is determined. This type of error minimization is mathematically solvable and is widely used in science and mathematics.

For a 4^{th} order polynomial equation, the least squares coefficients are obtained from the solution of the following matrix equation.

$$\begin{bmatrix} \mathbf{T} & \sum \mathbf{t} & \sum \mathbf{t}^2 & \sum \mathbf{t}^3 & \sum \mathbf{t}^4 \\ \sum \mathbf{t} & \sum \mathbf{t}^2 & \sum \mathbf{t}^3 & \sum \mathbf{t}^4 & \sum \mathbf{t}^5 \\ \sum \mathbf{t}^2 & \sum \mathbf{t}^3 & \sum \mathbf{t}^4 & \sum \mathbf{t}^5 & \sum \mathbf{t}^6 \\ \sum \mathbf{t}^3 & \sum \mathbf{t}^4 & \sum \mathbf{t}^5 & \sum \mathbf{t}^6 & \sum \mathbf{t}^7 \\ \sum \mathbf{t}^4 & \sum \mathbf{t}^5 & \sum \mathbf{t}^6 & \sum \mathbf{t}^7 & \sum \mathbf{t}^8 \end{bmatrix} \begin{bmatrix} \mathbf{a}_0 \\ \mathbf{b}_0 \\ \mathbf{c}_0 \\ \mathbf{d}_0 \\ \mathbf{e}_0 \end{bmatrix} = \begin{bmatrix} \sum \mathbf{p}(\mathbf{t}) \\ \sum (\mathbf{p}(\mathbf{t}) * \mathbf{t}) \\ \sum (\mathbf{p}(\mathbf{t}) * \mathbf{t}^2) \\ \sum (\mathbf{p}(\mathbf{t}) * \mathbf{t}^3) \\ \sum (\mathbf{p}(\mathbf{t}) * \mathbf{t}^4) \end{bmatrix}$$

where

$\mathbf{p}(\mathbf{T})$ is the current bar's price, $\mathbf{p}(\mathbf{T}-1)$ is the previous bar's price and $\mathbf{p}(\mathbf{1})$ is the price \mathbf{T} bars ago.

\mathbf{T} is the number of Bars in the Least Squares estimation

$\sum \mathbf{p}(\mathbf{t})$ is the summation of prices from $\mathbf{t}=1$ to \mathbf{T} bars

$\sum \mathbf{p}(\mathbf{t}) * \mathbf{t}$ is the summation of prices times \mathbf{t} from $\mathbf{t}=1$ to \mathbf{T} bars

$\sum \mathbf{t}$ is the summation of the integer \mathbf{t} from $\mathbf{t}=1$ to \mathbf{T} bars

$\sum \mathbf{t}^2$ is the summation of the integer \mathbf{t} squared from $\mathbf{t}=1$ to \mathbf{T} bars

etc.

Once the coefficients to the polynomial have been solved for we generate the forecast for the next bar's price which is given for the equation by:

$$\mathbf{P}_f = \mathbf{a}_0 + \mathbf{b}_0 * (\mathbf{T}+1) + \mathbf{c}_0 * (\mathbf{T}+1)^2 + \mathbf{d}_0 * (\mathbf{T}+1)^3 + \mathbf{e}_0 * (\mathbf{T}+1)^4$$

Where \mathbf{P}_f stands for price forecast.

With these coefficients, we can also generate the forecast for the next bar's *velocity* and *velocity* by the equations:

$$\mathbf{Velocity}(\mathbf{T}+1) = \mathbf{dP}_f / \mathbf{dt} = \mathbf{b}_0 + 2\mathbf{c}_0 * (\mathbf{T}+1) + 3\mathbf{d}_0 * (\mathbf{T}+1)^2 + 4\mathbf{e}_0 * (\mathbf{T}+1)^3$$

$$\mathbf{Velocity}(\mathbf{t}+1) = \mathbf{d}^2 \mathbf{P}_f / \mathbf{d}^2 \mathbf{t} = 2 \mathbf{c}_0 + 6\mathbf{d}_0 * (\mathbf{T}+1) + 12\mathbf{e}_0 * (\mathbf{T}+1)^2$$

We use the next bar forecast because changes in the trend are more quickly reflected in the forecast price, velocity and velocity than in the end point price, velocity and velocity.

Programs that solve for the solution to matrix equations can be found in the book "Numerical Recipes" by W. Press, et. al. However these type of matrix equation solvers are very slow and for these type of problems are unstable.

They cause numerical errors and floating point overflows due to matrix inversion ill conditioning which produces false results.

Fortunately these type of problems can be solved by a fast, efficient and accurate algorithm using Discrete

Appendix I: n^{th} Order Polynomial Next Bar's Forecast Math

Orthogonal Legendre Polynomials. This method is explained in detail in Norman Morrison's book entitled "Introduction to Sequential Smoothing and Prediction", Chapter 7 page 223., referenced at the end of this section.

Without going into detail here (see Morrison reference), the polynomial filter can now be represented by:

$$P_e(t) = \sum_{j=0}^n \beta_j \phi_j(t) \quad j=0 \text{ to } n$$

Where n is the polynomial order, T is the total number of Bars of data used in the Least Squares fit and

$$\beta_j = \sum_{k=0}^{T-1} p(t-T+k) \phi_j(k)$$

$\phi_j(t)$ = the *normalized* discrete Legendre polynomial. t = an integer from 0 to T

The coefficients, $\beta_0, \beta_1, \beta_2, \beta_3, \dots, \beta_n$ for a n^{th} order polynomial can now be solved for by the equation above and we can generate the forecast for the next bar's close, velocity and velocity which are given by the equations

$$P_F(T+1) = \beta_0 \phi_0(T+1) + \beta_1 \phi_1(T+1) + \beta_2 \phi_2(T+1) + \beta_3 \phi_3(T+1) + \dots + \beta_n \phi_n(T+1)$$

$$\text{Velocity} = (dP_F/dt)_{(T+1)} = \beta_1 (d\phi_1/dt)_{(T+1)} + \beta_2 (d\phi_2/dt)_{(T+1)} + \beta_3 (d\phi_3/dt)_{(T+1)} + \dots + \beta_n (d\phi_n/dt)_{(T+1)}$$

$$\text{Velocity} = (d^2P_F/d^2t)_{(T+1)} = \beta_2 (d^2\phi_2/d^2t)_{(T+1)} + \beta_3 (d^2\phi_3/d^2t)_{(T+1)} + \dots + \beta_n (d^2\phi_n/d^2t)_{(T+1)}$$

The n^{th} Order Fixed Memory Forecast Next Bar's Velocity Strategy Defined

The least squares forecast is constructed by solving for the least squares coefficients $\beta_1, \beta_2, \dots, \beta_n$ at each bar using the last T bars of closing prices and the Discrete Orthogonal Legendre Polynomial equations for β_j above. Then **Velocity** = $d^2P_F(T+1)/d^2t$ is constructed from the velocity equation above and plotted under the price chart. In general what we will be doing is following the plotted curve of **Velocity** which is calculated at each bar from the previous T bars. When the velocity is greater than a threshold amount *vup* we will go long. When the velocity is less than a threshold amount *-vdn* we will go short.

Buy Rule:

IF Velocity is greater than the threshold amount *vup* then buy at the market.

Sell Rule:

IF Velocity is less than the threshold amount *-vdn* then sell at the market.

References

1. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.

The Normalization Multiplier

What is the Multiplier ?

The n^{th} Order Fixed Memory Polynomial, also called an n^{th} Order Polynomial, is the least square fit of a polynomial of the form $\mathbf{b}_0 + \mathbf{b}_1 * \mathbf{t} + \mathbf{b}_2 * \mathbf{t}^2 + \mathbf{b}_3 * \mathbf{t}^3 + \dots + \mathbf{b}_n * \mathbf{t}^n$ to a *fixed* number of past data points. Where \mathbf{t} is discrete time bars. Time could be daily bars or one minute bars. We use the term “Fixed Memory” to designate that only a fixed number of data points are used to calculate the polynomial coefficients. It is assumed that the time bars occur at fixed intervals of time so tic bars would not be appropriate for this analysis. Least squares is a mathematical technique where the squared vertical distance between the data and the curve that is being fit to the data is minimized. When the net squared distance (also called the sum of the squared errors) is minimized, a unique set of coefficients $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n$ for the polynomial is determined. This type of error minimization is mathematically solvable and is widely used in science and mathematics. Once the \mathbf{b}_n coefficients are found using a lookback period of \mathbf{T} bars to calculate the \mathbf{b}_n coefficients, then the next bar’s estimate ($\mathbf{T}+1$) of the n^{th} order polynomial velocity and acceleration can be easily found by the equations below.

$$\text{Velocity}(\mathbf{T}+1) = dP_f / dt = \mathbf{b}_0 + 2\mathbf{c}_0 * (\mathbf{T}+1) + 3\mathbf{d}_0 * (\mathbf{T}+1)^2 + 4\mathbf{e}_0 * (\mathbf{T}+1)^3 + \dots + \mathbf{n} * \mathbf{b}_n * (\mathbf{T}+1)^{n-1}$$

$$\text{Acceleration}(\mathbf{t}+1) = d^2P_f / d^2t = 2 \mathbf{c}_0 + 9\mathbf{d}_0 * (\mathbf{T}+1) + 12\mathbf{e}_0 * (\mathbf{T}+1)^2 + \dots + \mathbf{n} * (\mathbf{n}-1) * \mathbf{b}_n * (\mathbf{T}+1)^{n-2}$$

Please see the *n^{th} Order Fixed Memory Polynomial Next Bar’s Forecast Math* section for a more detailed explanation.

For any tradable, the inputs to the polynomial are the **polynomial degree(Order)** and the number or lookback bars \mathbf{N} (denoted by \mathbf{T} in equations above). When we plot the velocity or acceleration we notice that the amplitude, and the maximum and minimum values of the velocity or acceleration vary quite significantly with different degree and \mathbf{N} inputs.

Below is a table of the standard deviation(SD) of the 56340 calculated Velocity values for different **degree** and \mathbf{N} inputs. We used 1min bars of the E-Mini from 8/1/2014 to 2/20/2015 to generate this table. As one can see the standard deviation of the velocity for each degree and \mathbf{N} vary greatly. For instance for degree=4, $\mathbf{N}=20$ the SD is 6.8 times the SD for degree=1, $\mathbf{N}=20$. This creates problems when trying to determine the correct ranges for vup/vdn and aup/adn during optimization.

@ES.D 1 min bars Date Range 1140801 to 1150220

Total Number of Bars=56340 Norm=0

FixmVn Multiplier= 1/SD to Scale Velocity pw and N Range to One SD

Degree	N	SD	1/SD
1	20	0.1902	5.2565
1	30	0.1540	6.4916
1	40	0.1328	7.5279
1	50	0.1183	8.4502
1	60	0.1077	9.2880
1	70	0.0996	10.0440
avg		0.1338	7.8430
2	20	0.4351	2.2982
2	30	0.3443	2.9046
2	40	0.2936	3.4060
2	50	0.1583	3.8275
2	60	0.2371	4.2180
2	70	0.2173	4.6010
avg		0.2981	3.5425
3	20	0.7854	1.2732

The Normalization Multiplier

3	30	0.5933	1.6855
3	40	0.4973	2.0111
3	50	0.4347	2.3005
3	60	0.3949	2.5324
3	70	0.3656	2.7352
avg		0.5119	2.0897
4	20	1.2924	0.7738
4	30	0.9279	1.0777
4	40	0.7582	1.3189
4	5	0.6542	1.5285
4	60	0.5804	1.7228
4	70	0.5314	1.8818
avg		0.7908	1.3839

The problem may get worse when we want to find good inputs for other tradables. Other tradables, because of their scales and tick size have much different Velocity ranges than the E-Mini for the same degree and N. Thus the NS search ranges have to be different for each different tradable.

To solve this problem and to have a standard search space for each tradable, I created a **Mult** input for each FixmXVA Velocity and Acceleration strategy and indicator. If each tradable's Velocity is multiplied by a number such that the standard deviation of that tradable's Velocity is around one, then the search space for vup and vdn for each tradable would be 0 to 3.5 SDs and we wouldn't have to change the TS search space every time we wanted to examine a new stock or future. The complicated equations that I use to normalize the ranges to one standard deviation were derived using the software TableCurve 3D, automated surface and equation discovery.